Responsible Decision-Making in the Face of Severe Uncertainty

Moshe Sniedovich

Department of Mathematics and Statistics
The University of Melbourne
moshe@moshe-online.com
www.moshe-online.com

USYD Seminar
May 16, 2008

Abstract

- How do you make responsible (robust?!) decisions in the face of severe uncertainty?
    - Classical decision theory
    - Robust optimization
    - Voodoo decision theory
    - Info-Gap decision theory (count fingers)
- Strong Australian flavor
- Ongoing campaign

Alternative title

If it looks too good to be true, it is . . .

This is a

Maths Classification G

presentation.

Maths Classification MA +18

versions can be found at

moshe-online.com

Programme

A bit of history

- First encounter: An invitation to a seminar (3/8/03)
- Second encounter: Seminar (Ben-Haim, 2/9/03).
- Requests for comments on Info-Gap: 2/9/03 – present.
- Informal critique: 3/9/03 – present.
- Formal critique: 1/12/06 – present.
- Campaign launch: 31/12/06.
- On the agenda:
  - Seminars
  - Honours theses
  - Conference presentations
  - Articles
  - WIKIPEDIA
  - Book
- Strongly interested in collaboration.

Executive Summary

- Decision-making under severe uncertainty is difficult.
- This is a very active area of research/practice.
- The Robust Optimization literature is very relevant.
- The Operations Research literature is very relevant.
- The Decision Theory literature is very relevant.
- The generic Info-Gap model is a simple vanilla instance of the classical Maximin Model [1945].
- Info-Gap is fundamentally flawed and is not suitable for decision-making under severe uncertainty.
- Practicing Info-Gap amounts to Voodoo decision-making.
- Reassessment of the use and promotion of Info-Gap in Australia is long overdue.

Motivation

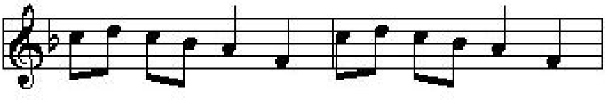
Opening paragraph of an on-line article on the FloodRiskNet website in the UK:

Hall and Ben-Haim, 2007, p. 1

Making Responsible Decisions (When it Seems that You Can't)
Engineering Design and Strategic Planning Under Severe Uncertainty

What happens when the uncertainties facing a decision maker are so severe that the assumptions in conventional methods based on probabilistic decision analysis are untenable?Jim Hall and Yakov Ben-Haim describe how the challenges of really severe uncertainties in domains as diverse as climate change, protection against terrorism and financial markets are stimulating the development of quantified theories of robust decision making.

Classical Decision Theory



Eg.

620-262: Decision Making

A Simple Problem

Good morning Sir/Madam:

I left on your doorstep four envelopes. Each contains a sum of money. You are welcome to open any one of these envelopes and keep the money you find there.

Please note that as soon as you open an envelope, the other three will automatically self-destruct, so think carefully about which of these envelopes you should open.

To help you decide what you should do, I printed on each envelope the possible values of the amount of money (in Australian dollars) you may find in it. The amount that is actually there is equal to one of these figures.

Unfortunately the entire project is under severe uncertainty so I cannot tell you more than this.

Good luck!

Joe.

So What Do You do?

Example

Envelope	*Possible Amount (Australian dollars)*
$E1$	$20, 10, 300, 786$
$E2$	$2, 40000, 102349, 5000000, 99999999, 56435432$
$E3$	$201, 202$
$E4$	200

Vote!

Modeling and Solution

- What is a decision problem ?
- How do we model a decision problem?
- How do we solve a decision problem?

Decision Tables

Think about your problem as a table, where

- rows represents decisions
- columns represent the relevant possible states of nature
- entries represent the associated payoffs/rewards/costs

Example

Env	Possible Amount ($AU)				
$E1$	20	10	300	786	
$E2$	2	4000000	102349	500000000	56435432
$E3$	201	202			
$E4$	200				

Classification of Uncertainty

Classical decision theory distinguishes between three levels of uncertainty regarding the state of nature, namely

- Certainty
- Risk
- Strict Uncertainty

Terminology:

$$
\begin{aligned}
\text{Strict Uncertainty} &\equiv \text{Severe Uncertainty}\\
&\equiv \text{Ignorance}\\
&\equiv \text{True Uncertainty}\\
&\equiv \text{Knightian Uncertainty}\\
&\equiv \text{Deep}\\
&\equiv \text{Extreme}
\end{aligned}
$$

Severe Uncertainty

Classical decision theory offers two basic principles for dealing with severe uncertainty, namely

- Laplace's Principle (1825)
- Wald's Principle (1945)

Conceptually:

Certainty $\xLeftarrow{\text{Wald}}$ **Severe Uncertainty** $\xRightarrow{\text{Laplace}}$ Risk

$\Downarrow$

Voodoo Decision Theory
Info-Gap Decision Theory
???

Bottom line: under severe uncertainty the estimate we have is a poor indicator of the true value it represents and is likely to be substantially wrong.

Laplace's Principle of Insufficient Reason (1825)

Assume that all the states are equally likely, thus use a uniform distribution function (μ) on the state space and regard the problem as decision-making under risk.

Laplace's Decision Rule

$$\max_{d\in\mathbb{D}} \int_{s\in S(d)} r(s,d)\mu(s)\,ds \qquad \text{Continuous case}$$

$$\max_{d\in\mathbb{D}} \frac{1}{|S(d)|} \sum_{s\in S(d)} r(s,d) \qquad \text{Discrete case}$$

Wald's Maximin Principle (1945)

Inspired by Von Neumann's [1928] Maximin model for 0-sum, 2-person games: Mother Nature is and adversary and is playing against you, hence apply the worst-case scenario. This transforms the problem into a decision-making under certainty.

Wald's Maximin Rule

$$\overset{\text{You!}}{\max_{d\in\mathbb{D}}} \ \overset{\text{Mama}}{\min_{s\in S(d)}} \ f(d,s)$$

Historical perspective: William Shakespeare (1564-1616)

> *The gods to-day stand friendly, that we may,*
> *Lovers of peace, lead on our days to age!*
> *But, since the affairs of men rests still incertain,*
> *Let's reason with the worst that may befall.*
>
> *Julius Caesar, Act 5, Scene 1*

Laplace vs Wald

Example

Env	*Possible Amount ($AU)*				
$E1$	20	10	300	786	
$E2$	2	4000	102349	50000	56435
$E3$	201	202			
$E4$	200				

Example

Env	*Possible Amount ($AU)*					*Laplace*	*Wald*
$E1$	20	10	300	786		279	10
$E2$	2	4000	10234	50000	56435	24134.2	2
$E3$	201	202				201.5	201
$E4$	200					200	200

Laplace vs Wald

Example

Env	*Possible Amount ($AU)*					*Laplace*	*Wald*
$E1$	20	10	300	786		279	10
$E2$	2	4000	10234	50000	56435	24134.2	2
$E3$	201	202				201.5	201
$E4$	200					200	200

Maximin

Perspective

- Used extensively because it ... gets rid of uncertainty!
- The "vanilla" version is often too "conservative".
- It is claimed that Savage's Minimax Regret is more DM-friendly.
- There are no general purpose algorithms in this area.
- Some Maximin problems are easy some are difficult.
- There are subtle modeling issues!
- Equivalent Mathematical Programming formulation

$$\max_{d\in D}\ \min_{s\in S(d)} f(d,s) = \max_{\substack{d\in D\\ v\in\mathbb{R}}}\{v : v \le f(d,s), \forall s \in S(d)\}$$

Severe Uncertainty

Warning!

- For obvious reasons, methodologies for decision-making under severe uncertainty are austere.
- There are no miracles in this business.
- The essential difficulty is: how do you sample the uncertainty region?
- The best estimate we have is very poor and likely to be substantially wrong.
- If you are offered a methodology that is too good to be true,. . . it is too good to be true!

Robust Optimization

WIKIPEDIA

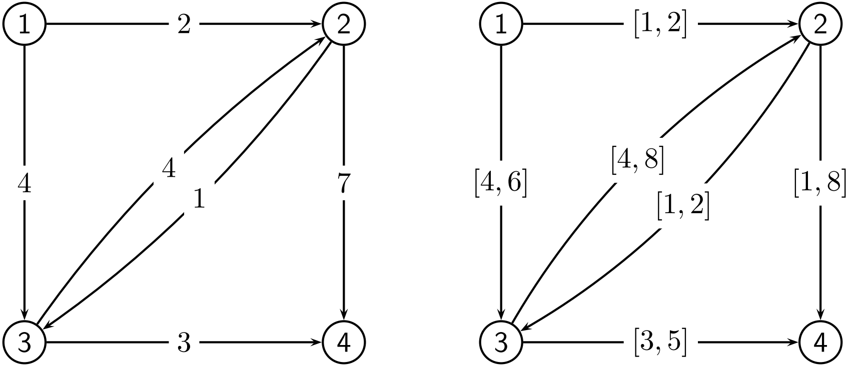
Robustness is the quality of being able to withstand stresses, pressures, or changes in procedure or circumstance. A system, organism or design may be said to be "robust" if it is capable of coping well with variations (sometimes unpredictable variations) in its operating environment with minimal damage, alteration or loss of functionality.

- Applies to both (known) variability and uncertainty
- Origin (in OR/MS): 1970s
- A very "hot" area of research these days ...
- See bibliography

Robust Optimization

Simple Example

Shortest path problem with variable arc lengths



Robust Optimization

Classification

- Robust Satisficing
  Robustness with respect to constraints of a satisficing problem or an optimization problem.
- Robust Optimizing
  Robustness with respect to the objective function of an optimization problem.
- Robust optimizing and satisficing
  Robustness with respect to both the objective function and constraints of an optimization problem.

Dominated by Wald's Maximin models and Savage's Minimax Regret models.

Robust Optimization

Classification

- Robust Satisficing

$$\text{Problem } P(u), u \in U:$$
$$\text{Find an } x \in X \text{ such that } g(x, u) \in C$$

- Robust Optimizing

$$\text{Problem } P(u), u \in U:$$
$$z^* := \operatorname*{opt}_{x \in X} f(x, u)$$

- Robust optimizing and satisficing

$$\text{Problem } P(u), u \in U:$$
$$z^* := \operatorname*{opt}_{x \in X(u)} f(x, u)$$

Robust Optimization

Robustness á la Maximin

- Robust Optimizing

$$z^* := \max_{d\in\mathbb{D}} \min_{s\in S(d)} f(d,s)$$

- Robust Satisficing

$$z^* := \max_{d\in\mathbb{D}} \min_{s\in S(d)} \varphi(d,s) := \begin{cases} \beta(d) & , \quad g(d,s)\in C \\ -\infty & , \quad g(d,s)\notin C \end{cases}$$

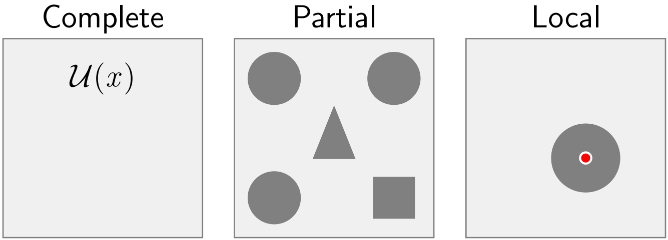
- Robust optimizing and satisficing

$$z^* := \max_{d\in\mathbb{D}} \min_{s\in S(d)} \psi(d,s) := \begin{cases} \gamma(d,s) & , \quad g(d,s)\in C \\ -\infty & , \quad g(d,s)\notin C \end{cases}$$

Robust Optimization

Degree of Robustness

- Complete
  $\forall u \in \mathcal{U}(x)$ (very conservative)
- Partial
  $\forall u \in U(x) \subseteq \mathcal{U}(x)$
- Local
  $\forall u \in U(x, \tilde{u}) \subseteq \mathcal{U}(x)$



Robust Optimization

Robustness á la Maximin

- Complete robustness

$$z^* := \max_{d\in\mathbb{D}} \min_{s\in S(d)} f(d,s)$$

- Partial robustness

$$z^* := \max_{\substack{d\in\mathbb{D}\\ U\subseteq S(d)}} \min_{s\in U} g(d,U,s)$$

where

$$g(d,U,s) := \begin{cases} \rho(U) & , \quad f(d,s) \geq f^*(s) \\ 0 & , \quad \text{otherwise} \end{cases}$$

Robust Optimization

Robustness á la Maximin

Local robustness

$$z^* := \max_{\substack{d \in \mathbb{D} \\ \alpha \geq 0}} \min_{s \in U(\alpha, \tilde{s})} g(d, \alpha, s)$$

where

$$g(d, \alpha, s) := \begin{cases} \alpha & , \quad f(d, s) \geq c \\ 0 & , \quad \text{otherwise} \end{cases}$$

Remark:

This approach is not suitable for severe uncertainty.

Voodoo Decision Theory

Encarta online Encyclopedia

Voodoo n

1. A religion practiced throughout Caribbean countries, especially Haiti, that is a combination of Roman Catholic rituals and animistic beliefs of Dahomean enslaved laborers, involving magic communication with ancestors.
2. Somebody who practices voodoo.
3. A charm, spell, or fetish regarded by those who practice voodoo as having magical powers.
4. A belief, theory, or method that lacks sufficient evidence or proof.

Voodoo

Voodoo

Voodoo

Voodoo

Voodoo

Voodoo Decision Theory

Apparently very popular,

Example

The behavior of Kropotkin's cooperators is something like that of decision makers using Jeffrey expected utility model in the Max and Moritz situation. Are ground squirrels and vampires using voodoo decision theory?

Brian Skyrms
Evolution of the Social Contract
Cambridge University Press, 1996.

Issue:

Evidential dependence, but causal independence.

The legend

An old legend has it that an ancient treasure is hidden in an Asian-Pacific island.

You are in charge of the treasure hunt. How would you plan the operation?

The legend

Main issue: location, location, location!

Terminology

Certainty

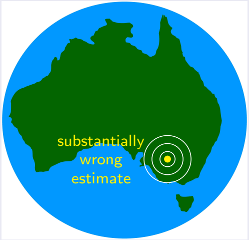
Risk

Severe Uncertainty

Voodooism

The Fundamental Theorem of Voodoo Decision Making

≈



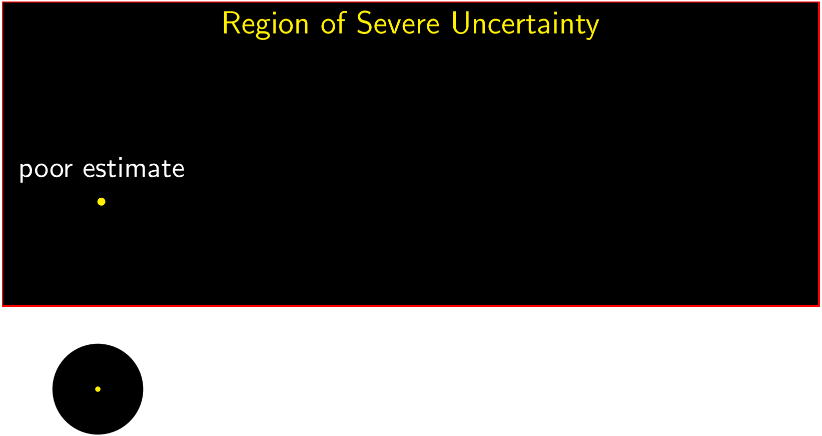
Severe Uncertainty

1·2·3 Recipe

1. Ignore the severe uncertainty.
2. Focus on the substantially wrong estimate you have.
3. Conduct the analysis in the immediate neighborhood of this estimate.

Voodooism

Voodoo Decision-Making



Voodooism

Voodoo Decision-Making

Just in case, . . . , the difficulty is that

Under SEVERE uncertainty

The estimate we use is

- A wild guess.
- A poor indication of the true value.
- Likely to be substantially wrong.

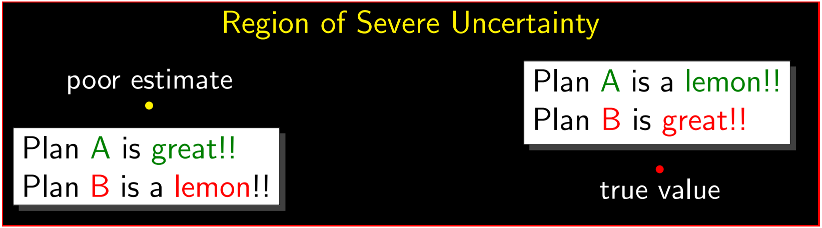
Hence,

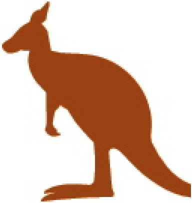
Beware!

Results obtained in the neighborhood of the estimate are likely to be substantially wrong in the neighborhood of the true value.

Voodooism

The Curse of Preference Reversal





VS

Voodooism

Summary

$$\text{GI} \longrightarrow \boxed{\mathfrak{Model}} \longrightarrow \text{GO}$$

$$\text{Wrong} \longrightarrow \boxed{\mathfrak{Model}} \longrightarrow \text{Wrong}$$

Info-Gap

Impressive Self-Portrait

> Info-gap decision theory is radically different from all current theories of decision under uncertainty. The difference originates in the modelling of uncertainty as an information gap rather than as a probability. The need for info-gap modeling and management of uncertainty arises in dealing with severe lack of information and highly unstructured uncertainty.
>
> Ben-Haim [2006, p. xii]

> In this book we concentrate on the fairly new concept of information-gap uncertainty, whose differences from more classical approaches to uncertainty are real and deep.
>
> Ben-Haim [2006, p. 11]

Info-Gap

Obvious Questions

1. Does Info-Gap substantiate these very strong claims?
2. Are these claims valid?

Not So Obvious Answers

1. No, it does not.
2. Certainly not.

It is therefore important to subject Info-Gap to a formal analysis – that actually should have been done seven years ago:

Info-Gap
Formal vs Analysis
Classical Decision Theory

Good news: should take no more than 5-10 minutes!

Info-Gap

Summary of Results

There are serious gaps in Info-Gap. The following is a partial list:

- Info-Gap has grave misconceptions about the state of the art in decision-making under severe uncertainty.
- Its generic decision model is a naive instance of the famous classical Maximin model (Wald, 1945).
- Its uncertainty model is fundamentally flawed. It does not deal with severe uncertainty, it simply ignores it.
- It is unsuitable for decision-making under severe uncertainty.
- There are other problematic issues with Info-Gap.

Generic Info-Gap Model

- Uncertainty region (set), $\mathfrak{U}$.
- A parameter u whose true value, u°, is unknown except that $u^\circ \in \mathfrak{U}$.
- An estimate $\tilde{u} \in \mathfrak{U}$ of u°.
- A parametric family of nested regions of uncertainty, $\mathcal{U}(\alpha, \tilde{u}) \subseteq \mathfrak{U}, \alpha \geq 0$, of varying size (α), centered at $\tilde{u}$. That is, it is assumed that $\mathcal{U}(0, \tilde{u}) = \{\tilde{u}\}$ and that $\mathcal{U}(\alpha, \tilde{u})$ is non-decreasing with α, namely

$$\mathcal{U}(\alpha, \tilde{u}) \subseteq \mathcal{U}(\alpha + \varepsilon, \tilde{u}) \ , \ \forall \varepsilon \geq 0 \tag{1}$$

- Set of feasible decisions, $\mathbb{Q}$.
- Reward function $R: \ \mathbb{Q} \times \mathfrak{U} \rightarrow \mathbb{R}$.
- Critical reward level, $r_c \in \mathbb{R}$.

Generic Info-Gap Model

Robustness of a decision

$$\hat{\alpha}(q, r_c) := \max \left\{ \alpha \geq 0 : r_c \leq \min_{u \in \mathcal{U}(\alpha, \tilde{u})} R(q, u) \right\} \tag{2}$$

Optimal robustness

$$\hat{\alpha}(r_c) := \max_{q \in \mathbb{Q}} \hat{\alpha}(q, r_c) \tag{3}$$

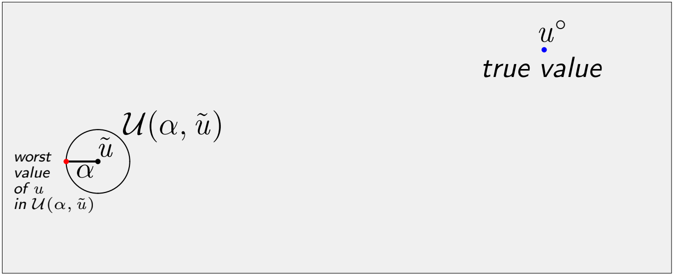
$$= \max_{q \in \mathbb{Q}} \max \left\{ \alpha \geq 0 : r_c \leq \min_{u \in \mathcal{U}(\alpha, \tilde{u})} R(q, u) \right\} \tag{4}$$

Generic Info-Gap Model

Complete Model

$$\hat{\alpha}(r_c) := \max_{q\in\mathbb{Q}} \max \left\{ \alpha \geq 0 : r_c \leq \min_{u\in\mathcal{U}(\alpha,\tilde{u})} R(q,u) \right\} \tag{5}$$

Region of Severe Uncertainty, $\mathfrak{U}$



Info-Gap

Complete Generic Model

$$\hat{\alpha}(r_c) := \max_{q \in \mathbb{Q}} \ \max \left\{ \alpha \geq 0 : r_c \leq \min_{u \in \mathcal{U}(\alpha, \tilde{u})} R(q, u) \right\} \qquad (6)$$

Fundamental FAQs

1. Is this new? Definitely not!
2. Is this radically different? Definitely not!
3. Does it make sense? Definitely not!

So what is all this hype about Info-Gap ?!

Good question!

Info-Gap

First Impression

Complete Generic Model

$$\hat{\alpha}(r_c) := \max_{q\in\mathbb{Q}} \max \left\{ \alpha \geq 0 : r_c \leq \min_{u\in\mathcal{U}(\alpha,\tilde{u})} R(q,u) \right\} \tag{7}$$

Observations

- This model does not deal with severe uncertainty, it simply and unceremoniously ignores it.
- The analysis is invariant with $\mathfrak{U}$: the same solution for all $\mathfrak{U}$ such that $\mathcal{U}(\hat{\alpha}(r_c), \tilde{u}) \subseteq \mathfrak{U}$.
- This model is fundamentally flawed.
- This model advocates voodoo decision-making.

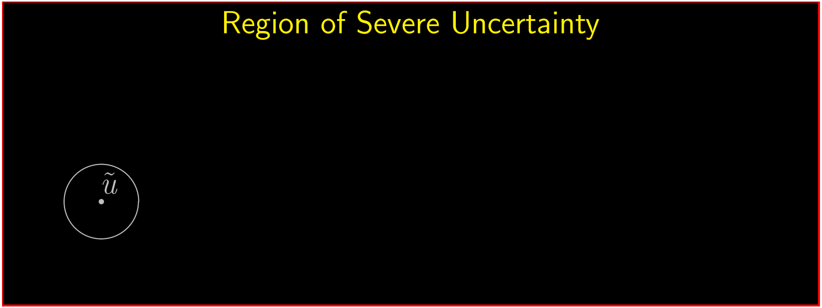
Info-Gap

First Impression

Fool-Proof Recipe

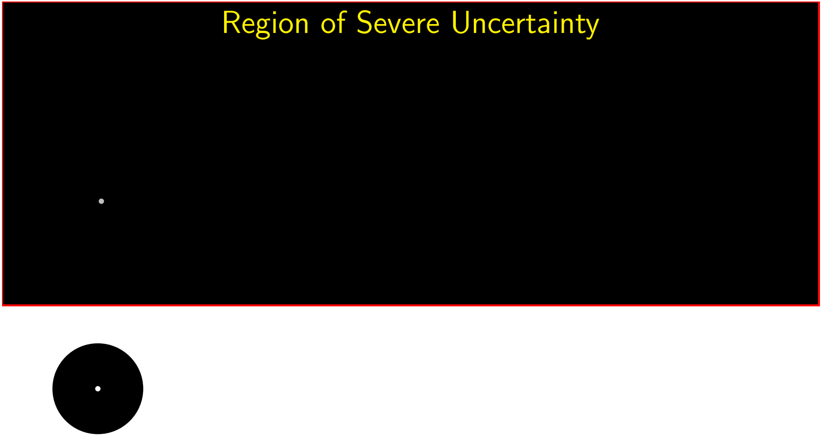
Step 1: *Ignore the severe uncertainty.*

Step 2: *Focus instead on the poor estimate and its immediate neighborhood.*



Info-Gap

First Impression



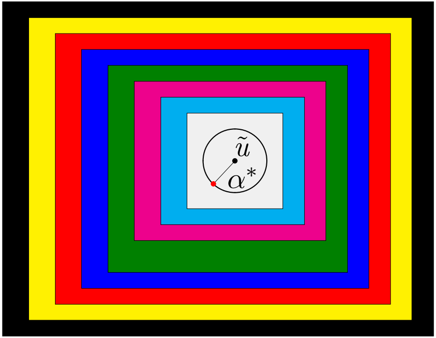
Recall that this is voodoo decision making!

Info-Gap

Complete Generic Model

$$\alpha^* := \max_{q \in \mathbb{Q}} \max \left\{ \alpha \geq 0 : r_c \leq \min_{u \in \mathcal{U}(\alpha, \tilde{u})} R(q, u) \right\} \qquad (8)$$

Fundamental Flaw



Info-Gap

More formally

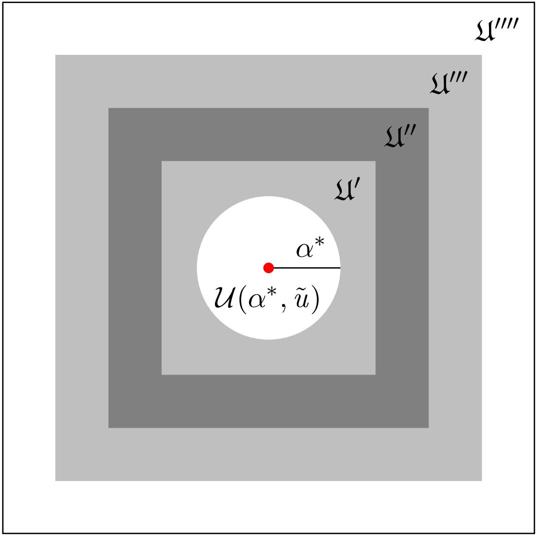
Theorem (Sniedovich, 2007)

Info-Gap's robustness model is invariant to the size of the total region of uncertainty $\mathfrak{U}$ for all $\mathfrak{U}$ larger than $\mathcal{U}(\alpha^*, \tilde{u})$, where $\alpha^* := \hat{\alpha}(r_c)$.
That is, the model yields the same results for all $\mathfrak{U}$ such that

$$\mathcal{U}(\alpha^* + \varepsilon, \tilde{u}) \subseteq \mathfrak{U} \ , \ \varepsilon > o$$

Info-Gap

Info-Gap's Invariance Property



Info-Gap

Theorem (Sniedovich 2007, 2008)

Info-Gap's robustness model is a simple instance of Wald's Maximin model. Specifically,

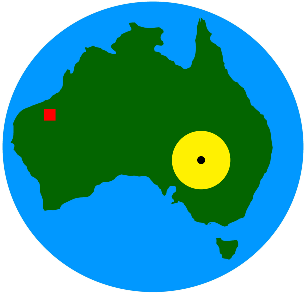
$$\begin{aligned}\alpha(q) :&= \max_{\alpha \geq 0} \left\{ \alpha : r_c \leq \min_{u \in \mathcal{U}(\alpha, \tilde{u})} R(q, u) \right\} \\ &= \max_{\alpha \geq 0} \min_{u \in \mathcal{U}(\alpha, \tilde{u})} \psi(q, \alpha, u)\end{aligned}$$

where

$$\psi(q, \alpha, u) := \begin{cases} \alpha \ , & r_c \leq R(q, u) \\ 0 \ , & r_c > R(q, u) \end{cases} \ , \alpha \geq 0, q \in \mathbb{Q}, u \in \mathcal{U}(\alpha, \tilde{u})$$

Info-Gap: Typical misconception

Treasure Hunt



- Region of uncertainty.
- Estimate of the location.
- Region affecting Info-Gap's analysis.
- True (unknown) location.

Hence, Info-gap may conduct its robustness analysis in the vicinity of Brisbane (QLD), whereas for all we know the true location of the treasure may be somewhere in the middle of the Simpson desert or perhaps in down town Melbourne (VIC). Perhaps.

Info-Gap: : Typical misconception

One finds in the Info-Gap literature numerous claims that Info-Gap theory answers questions such as this:

> *How wrong can the model and data be without jeopardizing the quality of the outcome?*

The correct interpretation of Info-Gap robustness is as follows:

> *The robustness of a decision is the maximum deviation from the given estimate such that the performance requirement is satisfied for every value of the parameter in the immediate neighborhood of the estimate stipulated by this deviation.*

Info-Gap: Typical misconception

- Info-gap's robustness tells us is how "safe" we are in the immediate neighborhood of the estimate.
- The trouble is, of course, that subject to severe uncertainty the estimate is a wild guess, . . . a poor . . . substantially wrong, . . . etc, etc, etc.
- It is a simple exercise to construct examples where a decision is highly robust in the neighborhood of the estimate, but fragile elsewhere in the total region of uncertainty, and vice versa.

Info-Gap: $$$$$$$

Eg.

2004 Competitive Research Grants (USA)

Title: . . .
Principal Investigator: . . .
Affiliation: . . .
Award: > $125,000

Research project based on info-gap . . .

Info-Gap: $$$$$$$

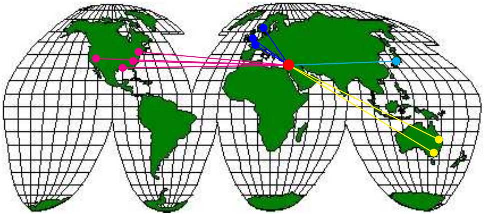
Competitive Research Grants (Australia)

AU$???

Info-Gap Enterprise

- Publications
- Research Grants
- Keynote Lectures
- Workshops
- PhDs

Info-Gap: Bird's View



Profile

- Financial institutions
- Research centers
- Universities
- Government agencies
- Australia is an international stronghold

Conclusions

- Decision-making under severe uncertainty is difficult.
- It is a thriving area of research/practice.
- The Robust Optimization literature is extremely relevant.
- The Decision Theory literature is extremely relevant.
- The Operations Research literature is very relevant.
- Info-Gap's decision model is neither new nor radically different.
- Info-Gap's uncertainty model is fundamentally flawed and unsuitable for decision-making under severe uncertainty.
- Info-Gap exhibits a severe information-gap about the state of the art in decision-making under severe uncertainty.
- It is time to reassess the use of Info-Gap in Australia.
- Join the Campaign
- Join the Research

Bibliography

- Ben-Haim, Y. 1996. *Robust Reliability in the Mechanical Science,* Springer Verlag.
- Ben-Haim, Y. 2001. *Information Gap Decision Theory.* Academic Press.
- Ben-Haim, Y. 2006. *Info-Gap Decision Theory.* Elsevier.
- Ben-Tal A. El Ghaoui, L. & Nemirovski, A. 2006. *Mathematical Programming*, Special issue on *Robust Optimization*107(1-2).
- Dembo, R.S. 1991. Scenario optimization. *Annals of Operations Research* 30(1): 63-80.
- Demyanov, V.M. and Malozemov, V.N. 1990. *Introduction to Minimax,* Dover.

- Du, D.Z. and Pardalos, P.M. 1995. *Minimax and Applications*, Springer Verlag.
- Eiselt, H.A., Sandblom, C.L. and Jain, N. 1998. A Spatial Criterion as Decision Aid for Capital Projects: Locating a Sewage Treatment Plant in Halifax, Nova Scotia, *Journal of the Operational Research Society*, 49(1), 23-27.
- Eiselt, H.A. and Langley A. 1990. Some extensions of domain criteria in decision making under uncertainty, *Decision Sciences*, 21, 138-153.
- Francis, R.L., McGinnis, Jr, L.F. & White, J.A. 1992. *Facility Layout and Location: An Analytical Approach.* Prentice Hall.
- French, S.D. 1988. *Decision Theory*, Ellis Horwood.

- Hall, J. & Ben-Haim, Y. 2007. Making Responsible Decisions (When it Seems that You Can't). www.floodrisknet.org.uk/a/2007/11/hall -benhaim.pdf.
- Kouvelis, P. & Yu, G. 1997. *Robust Discrete Optimization and Its Applications.*, Kluwer.
- Reemstem, R. and Rückmann, J. (1998). *Semi-Infinite Programming,* Kluwer, Boston.
- Resnik, M.D. 1987. *Choices: an Introduction to Decision Theory.* University of Minnesota Press: Minneapolis.
- Rosenhead M.J, Elton M, Gupta S.K. 1972. Robustness and Optimality as Criteria for Strategic Decisions, *Operational Research Quarterly*, 23(4), 413-430.

- Rustem, B. & Howe, M. 2002. *Algorithms for Worst-case Design and Applications to Risk Management.* Princeton University Press.
- Skyrms, B. 1996. *Evolution of the Social Contract,* Cambridge University Press.
- Sniedovich, M. 2007. The art and science of modeling decision-making under severe uncertainty. *Journal of Manufacturing and Services,* 1(1-2): 111-136.
- Sniedovich, M. 2008. Wald's Maximin Model: A Treasure in Disguise! *Journal of Risk Finance,* 9(3), in press.
- Starr, M.K. 1963. *Product design and decision theory,* Prentice-Hall, Englewood Cliffs, NJ.
- Starr, M. K. 1966. A Discussion of Some Normative Criteria for Decision-Making Under Uncertainty, *Industrial Management Review,* 8(1), 71-78.

- Tintner, G. 1952. Abraham Wald's contributions to econometrics. *The Annals of Mathematical Statistics* 23(1): 21-28.
- Vladimirou, H. & Zenios, S.A. 1997. Stochastic Programming and Robust Optimization. In Gal, T, & Greenberg H.J. (ed.), *Advances in Sensitivity Analysis and Parametric Programming.* Kluwer.
- von Neumann, J. 1928. Zur theories der gesellschaftsspiele, *Math. Annalen*, Volume 100, 295-320.
- von Neumann, J. and Morgenstern, O. 1944. *Theory of Games and Economic Behavior*, Princeton University Press.
- Wald, A. 1945. Statistical decision functions which minimize the maximum risk, *The Annals of Mathematics,* 46(2), 265-280.

Wald, A. 1950. *Statistical Decision Functions.* John Wiley.