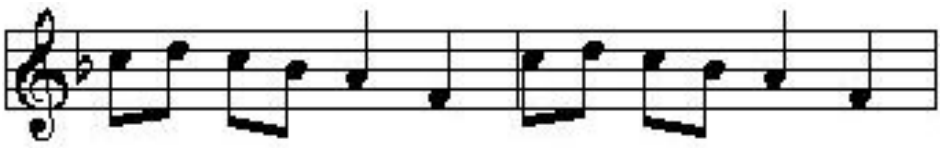
A Critique of Info-Gap
Myths and Facts

Moshe Sniedovich

Department of Mathematics and Statistics
The University of Melbourne
www.ms.unimelb.edu.au/∼moshe/



Second SRA Conference
August 20-21, 2007
Hobart, TAS, Australia

Admin matters

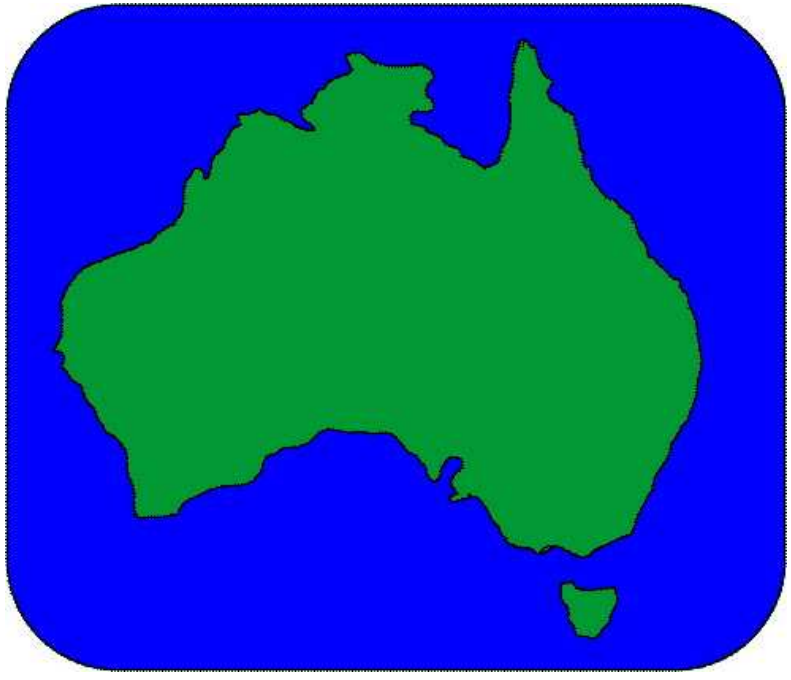
Maths Classification MA +18

10:20 – 10:40

Programme

The legend

An old legend has it that an ancient treasure is hidden in an Asian-Pacific island.

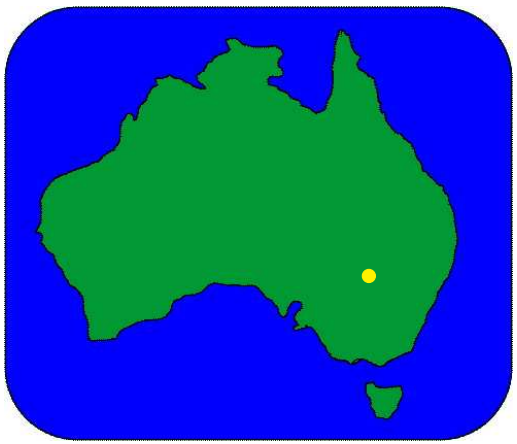


You are in charge of the treasure hunt. How would you plan the operation?

The legend

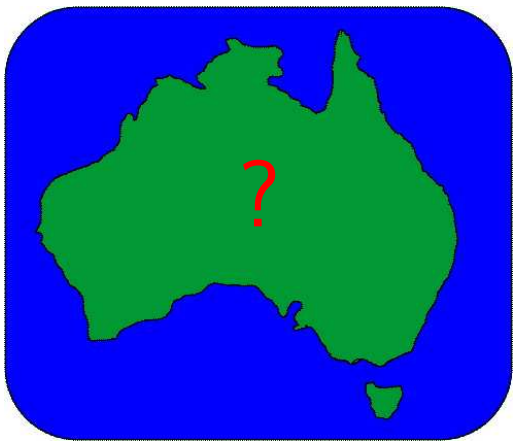
Main issue: location, location, location!

Terminology



Certainty

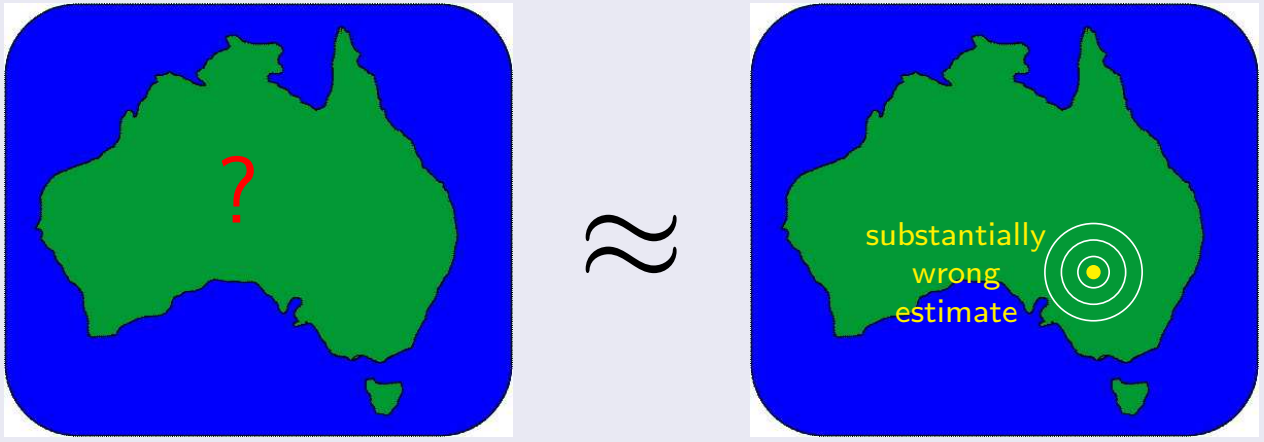
Risk



Severe Uncertainty

Voodooism

The Fundamental Theorem of Voodoo Decision Making



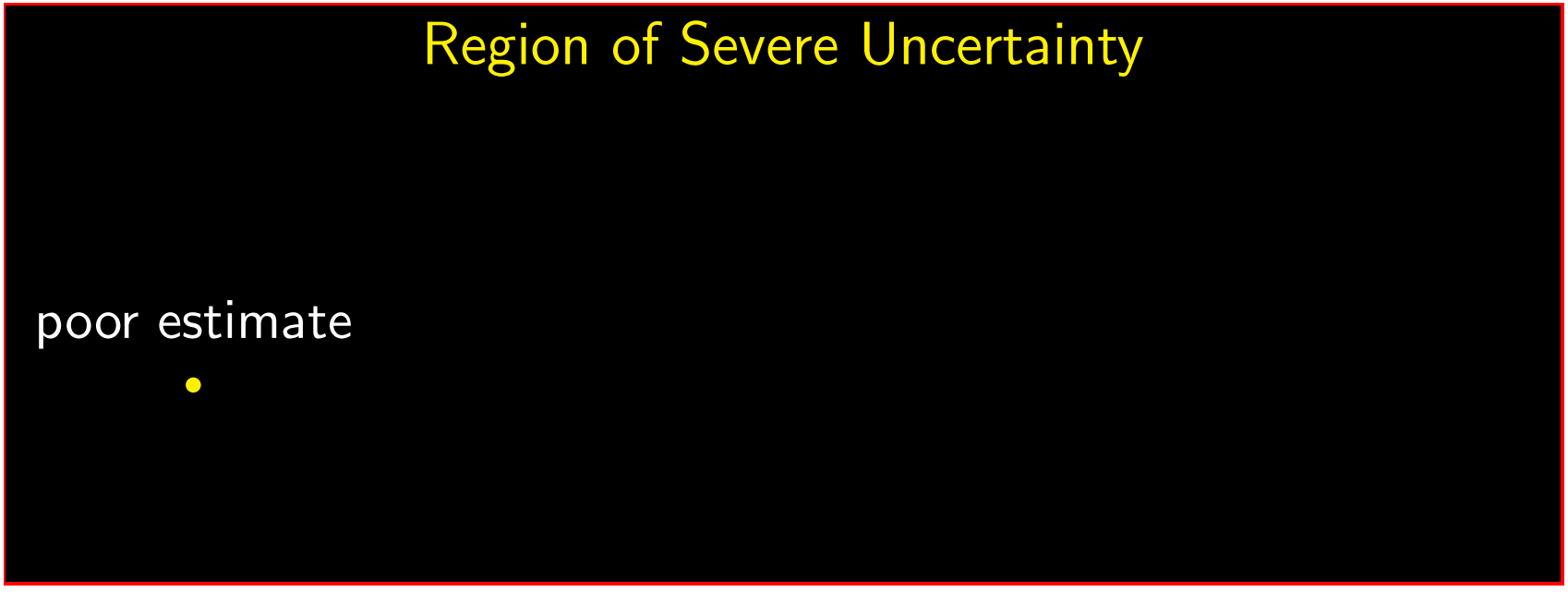
Severe Uncertainty

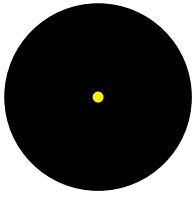
1·2·3 Recipe

1. Ignore the severe uncertainty.
2. Focus on the substantially wrong estimate you have.
3. Conduct the analysis in the immediate neighborhood of this estimate.

Voodooism

Voodoo Decision-Making





Voodooism

Voodoo Decision-Making

Just in case, . . . , the difficulty is that

Under SEVERE uncertainty

The estimate we use is

- " . . . a poor indication of the true value . . . "
- " . . . likely to be substantially wrong . . . "

Ben-Haim [2006, pp. 280-1]

Hence,

Beware!

Results obtained in the neighborhood of the estimate are likely to be substantially wrong in the neighborhood of the true value.

Voodooism

The Curse of Preference Reversal

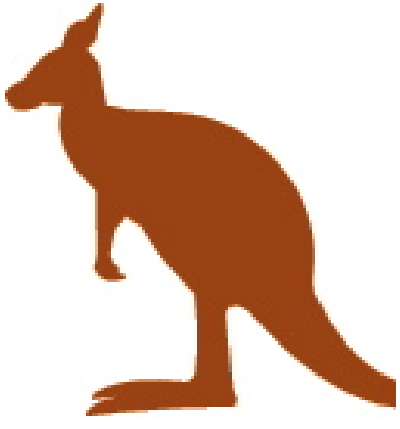
Region of Severe Uncertainty

poor estimate

Plan A is great!!
Plan B is a lemon!!

Plan A is a lemon!!
Plan B is great!!

true value



VS

Voodooism

Summary

$$\text{GI} \longrightarrow \boxed{\mathfrak{Model}} \longrightarrow \text{GO}$$

$$\text{Wrong} \longrightarrow \boxed{\mathfrak{Model}} \longrightarrow \text{Wrong}$$

Info-Gap

Self-Portrait

Info-gap decision theory is radically different from all current theories of decision under uncertainty. The difference originates in the modelling of uncertainty as an information gap rather than as a probability. The need for info-gap modeling and management of uncertainty arises in dealing with severe lack of information and highly unstructured uncertainty.

Ben-Haim [2006, p. xii]

In this book we concentrate on the fairly new concept of information-gap uncertainty, whose differences from more classical approaches to uncertainty are real and deep.

Ben-Haim [2006, p. 11]

Info-Gap

Obvious Questions

1. Does Info-Gap substantiate these very strong claims?
2. Are these claims valid?

Not So Obvious Answers

1. No, it does not.
2. Certainly not.

It is therefore important to subject Info-Gap to a formal analysis – that actually should have been done seven years ago:

Info-Gap

Formal vs Analysis

Classical Decision Theory

Good news: should take no more than 5-10 minutes!

Info-Gap

Summary of Results

There are serious gaps in Info-Gap. The following is a partial list:

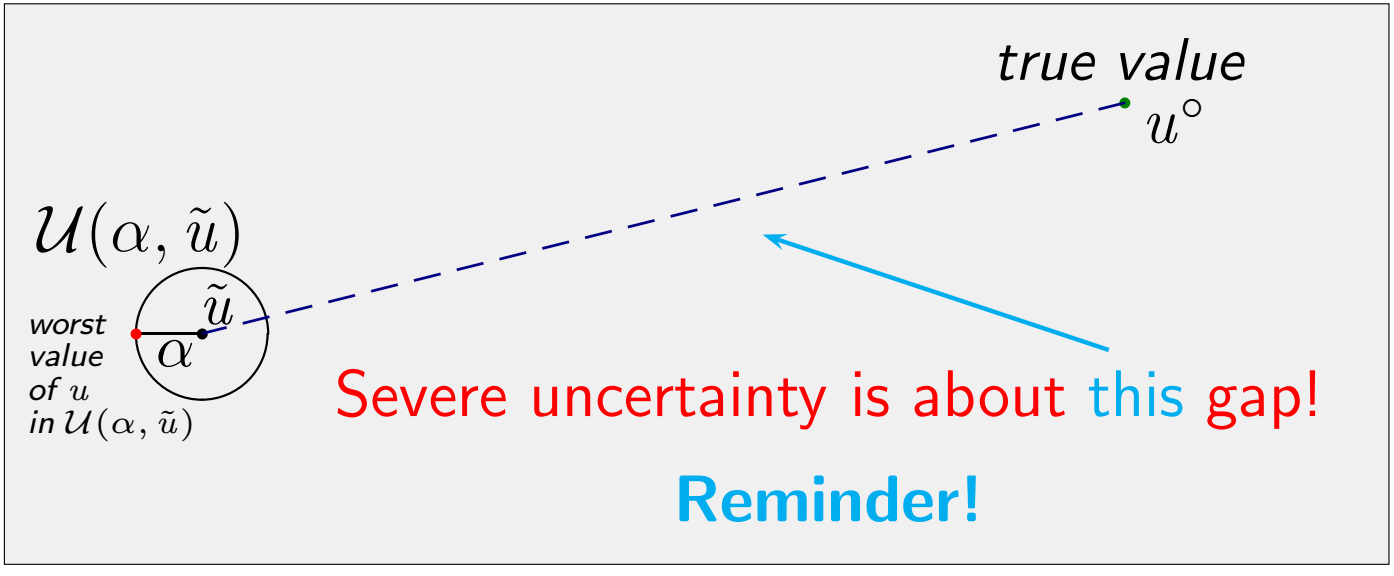
- Info-Gap has grave misconceptions about the state of the art in decision-making under severe uncertainty.
- Its generic decision model is a naive instance of the famous classical Maximin model (Wald, 1945).
- Its uncertainty model is fundamentally flawed. It does not deal with severe uncertainty, it simply ignores it.
- It is unsuitable for decision-making under severe uncertainty.
- There are other problematic issues with Info-Gap.

Generic Info-Gap Model

Complete Generic Model

$$\hat{\alpha}(r_c) := \max_{q \in \mathbb{Q}} \max \left\{ \alpha \geq 0 : r_c \leq \min_{u \in \mathcal{U}(\alpha, \tilde{u})} R(q, u) \right\} \qquad (1)$$

Region of Severe Uncertainty, $\mathcal{U}$



Generic Info-Gap Model

Generic Model

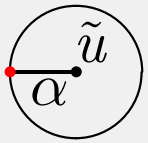
$$\hat{\alpha}(q, r_c) := \max \left\{ \alpha \geq 0 : r_c \leq \min_{u \in \mathcal{U}(\alpha, \tilde{u})} R(q, u) \right\} \qquad (2)$$

Fundamental Conceptual Flaw

Decisions are ranked according to their robustness in the neighborhood of a substantially wrong estimate.

Region of Severe Uncertainty, $\mathfrak{U}$

Substantially wrong estimate



true value u°

Info-Gap

Complete Generic Model

$$\hat{\alpha}(r_c) := \max_{q \in \mathbb{Q}} \max \left\{ \alpha \geq 0 : r_c \leq \min_{u \in \mathcal{U}(\alpha, \tilde{u})} R(q, u) \right\} \quad (3)$$

Fundamental FAQs

1. Is this new? Definitely not!
2. Is this radically different? Definitely not!
3. Does it make sense under severe uncertainty? Definitely not!

So what is all this fuss about Info-Gap ?!

Good question!

Myths and Facts

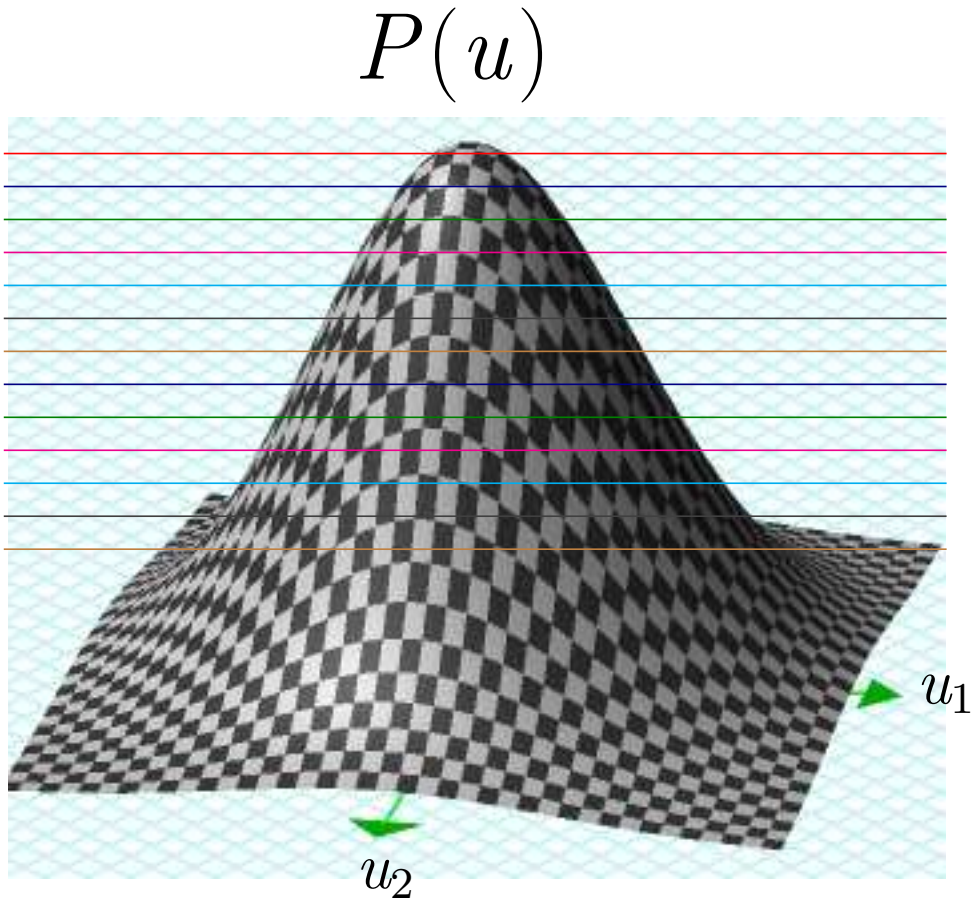
Myth # 1

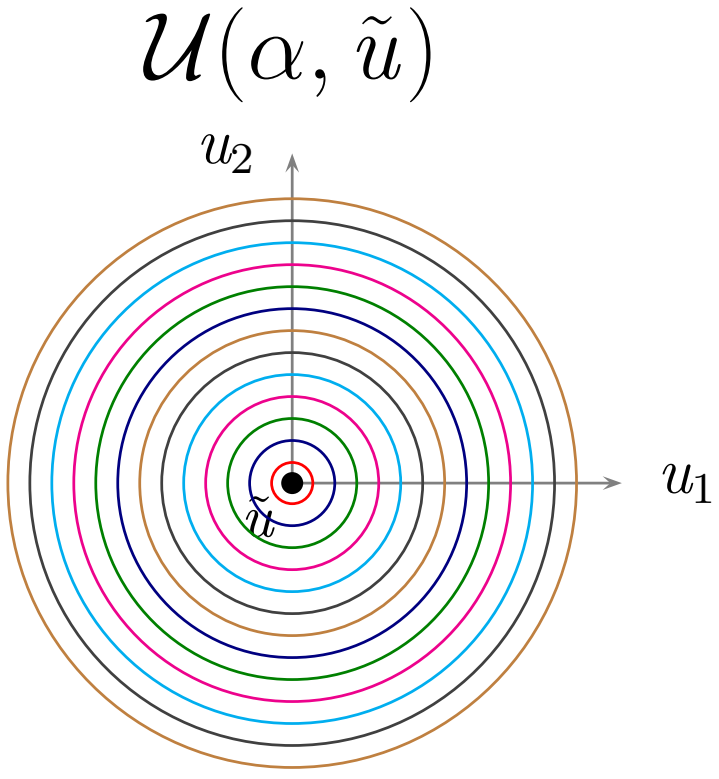
Info-Gap's uncertainty model is inherently non-probabilistic in nature.

Fact # 1

- This is an illusion.
- Info-Gap's uncertainty model is a typical subjective probability model.
- The boundaries of the regions of uncertainty represent contours of a subjective probability function of u (Sniedovich [2006] for technical details).

Myths and Facts





Different representations of the same thing.
Sniedovich [2006]

Myths and Facts

Myth # 2

Classical decision theory does not offer probability-free approaches to decision-making under severe uncertainty.

Fact # 2

- This is astonishing!
- Demonstrates a severe lack of familiarity with decision theory.
- Practically all introductory textbooks on decision theory discuss such approaches.
- The most famous one is Wald's Maximin Model [1945].

Example

CHOICES

AN INTRODUCTION TO DECISION THEORY

Michael D. Resnik

1987

Chapter 1

Introduction

1-1 What is Decision Theory?

1-2 The Basic Framework

1-3 Certainty, Ignorance, and Risk

1-4 Decision Trees

1-5 References

Example

Chapter 2

Decisions Under Ignorance

2-1 Preference Ordering

2-2 The Maximin Rule

2-3 The Minimax Regret Rule

2-4 The Optimism-Pessimism Rule

2-5 The Principle of Insufficient Reason

2-6 Too many Rules?

2-7 An application in Social Philosophy

2-8 References

Example

Chapter 3

Decisions Under Risk: Probability

3-1 Maximizing Expected Values

3-2 Probability Theory

2-3 Interpretations of Probability

2-8 References

Chapter 4

Decisions under Risk: Utility

4-1 Interval Utility Scales

...

Myths and Facts

Myth # 3

Info-Gap is a new theory that is radically different from all current theories for decision-making under severe uncertainty (Ben-Haim [2001, 2006])

Fact # 3

- Info-Gap's generic model is neither new nor radically different.
- It is a simple instance of Wald's Maximin model [1945] (Sniedovich [2006])

Maximin [1945]	*Info-Gap*
$\max_{d\in\mathbb{D}} \min_{s\in S_d} f(d,s)$	$\max_{q\in\mathbb{Q}} \max \left\{ \alpha \geq 0 : r_c \leq \min_{u\in\mathcal{U}(\alpha,\tilde{u})} R(q,u) \right\}$

Myths and facts

Theorem (Sniedovich [2006])

Info-Gap's generic model is a simple Maximin Model.

Proof.

$$\alpha(r_c) := \max_{q\in\mathbb{Q}} \max \left\{ \alpha \geq 0 : r_c \leq \min_{u\in\mathcal{U}(\alpha,\tilde{u})} R(q,u) \right\} \tag{4}$$

$$= \max_{q\in\mathbb{Q},\alpha\geq 0} \min_{u\in\mathcal{U}(\alpha,\tilde{u})} \overbrace{\alpha \cdot (r_c \preceq R(q,u))}^{f(q,\alpha,u)} \tag{5}$$

$$a \preceq b := \begin{cases} 1 & , \quad a \leq b \\ 0 & , \quad a > b \end{cases} \tag{6}$$

□

Myths and Facts

Myth # 4

Info-Gap deals with severe uncertainty.

Fact # 4

Info-Gap does not deal with severe uncertainty. It ignores it.
This involves:

- Replacing severe uncertainty with a very poor estimate of the parameter under consideration.
- Conducting a standard maximin analysis in the neighborhood of this very poor estimate.

Myths and Facts

Myth # 5

Info-Gap models address basic questions such as:

- "How wrong can the models and data be, without jeopardizing the quality of the outcome?"
- "How wrong can a model and its parameters be before jeopardizing the quality of decisions made on the basis of this model?"
- "How wrong could this model be, before I should change my decision?"

Fact # 5

- Info-Gap definitely does not address these questions.
- Indeed, it is misleading to contend that this is so.
- We know nothing about the quality of the decision.

Myth and Facts

The Issues

- The fine-print is dangerously missing here!
- Consult your lawyer to avoid a catastrophe!

Fine Print

Info-Gap asks the following intriguing question

How wrong can a parameter u of a model be before jeopardizing the quality of decisions made on the basis of this parameter – given that

1. $\tilde{u}$ is a good, accurate estimate.
2. $\tilde{u}$ is a poor estimate, that can be substantially wrong.

1. To make sense, scientifically, and stay out of jail.
2. To adequately represent severe uncertainty.

Myths and Facts

This myth is remarkable!

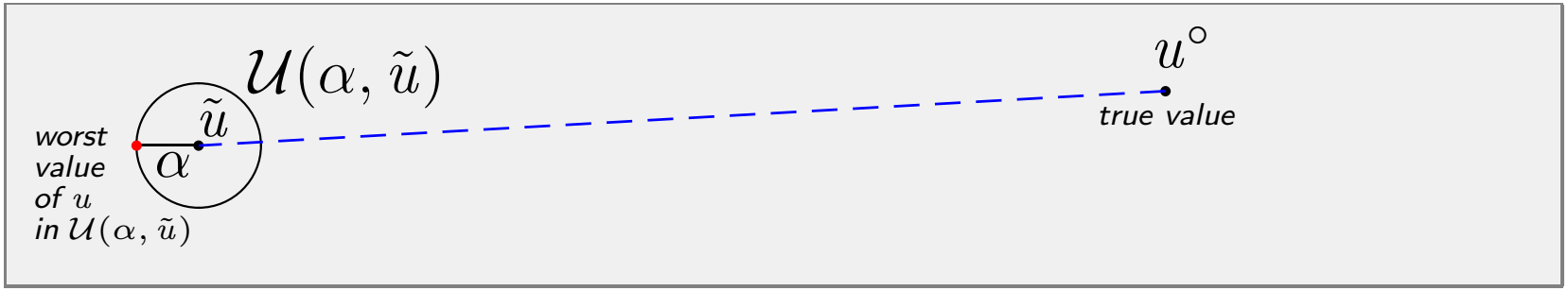
It is an exercise in wishful thinking:

- You acknowledge that $\tilde{u}$ is " . . . a poor indication of the true value of u . . . " and that " . . . it is likely to be substantially wrong. . . "
- Yet, you knowingly deceive yourself, pretending that the model based on $\tilde{u}$ is fine, and your only concern is how far you can deviate from it without running into trouble.

Myths and Fact

Bad news . . .

You were already in deep trouble before you started deviating, mate!



Indeed, you may well be better off deviating from the poor estimate!

Correction:

$$\alpha(q, r_c) := \max \left\{ \alpha \geq 0 : r_c \leq \min_{u \in \mathcal{U}(\alpha, \tilde{u})} R(q, u) \right\}$$

Info-Gap Interpretation

$\hat{\alpha}(q, r_c)$:= robustness of decision q given the required reward r_c.

Correct interpretation

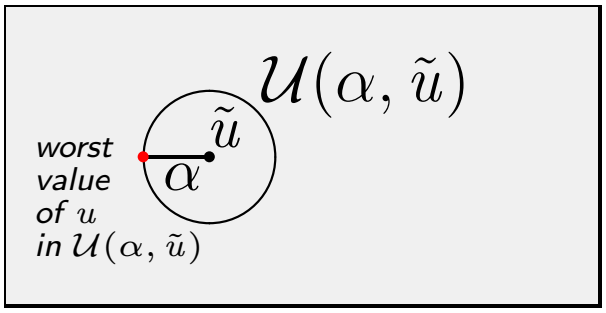
$\hat{\alpha}\left(q, r_c, |\tilde{u}\right)$:= robustness of decision q given the required reward r_c, in the neighborhood of the POOR estimate $\tilde{u}$ that is likely to be SUBSTANTIALLY WRONG.

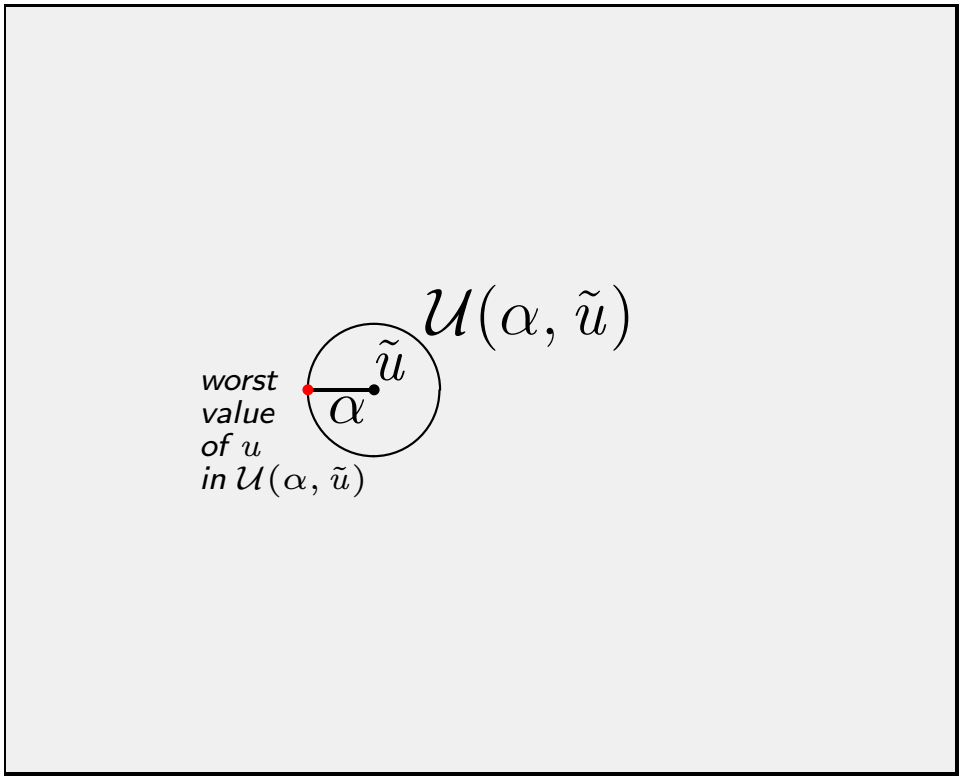
Note the analogy with the distinction between marginal and conditional probabilities!

Myth and Facts

Observation

The Info-Gap analysis is invariant with the actual size of the total region of uncertainty, $\mathfrak{U}$.
Surely, this makes no sense.



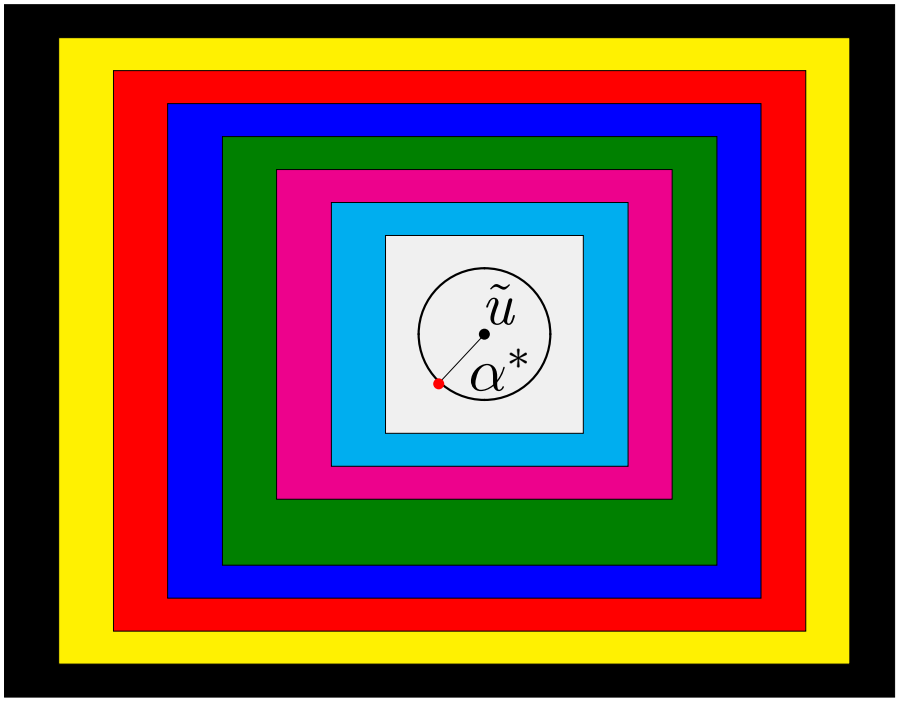


Info-Gap

Complete Generic Model

$$\alpha^* := \max_{q \in \mathbb{Q}} \max \left\{ \alpha \geq 0 : r_c \leq \min_{u \in \mathcal{U}(\alpha, \tilde{u})} R(q, u) \right\} \qquad (7)$$

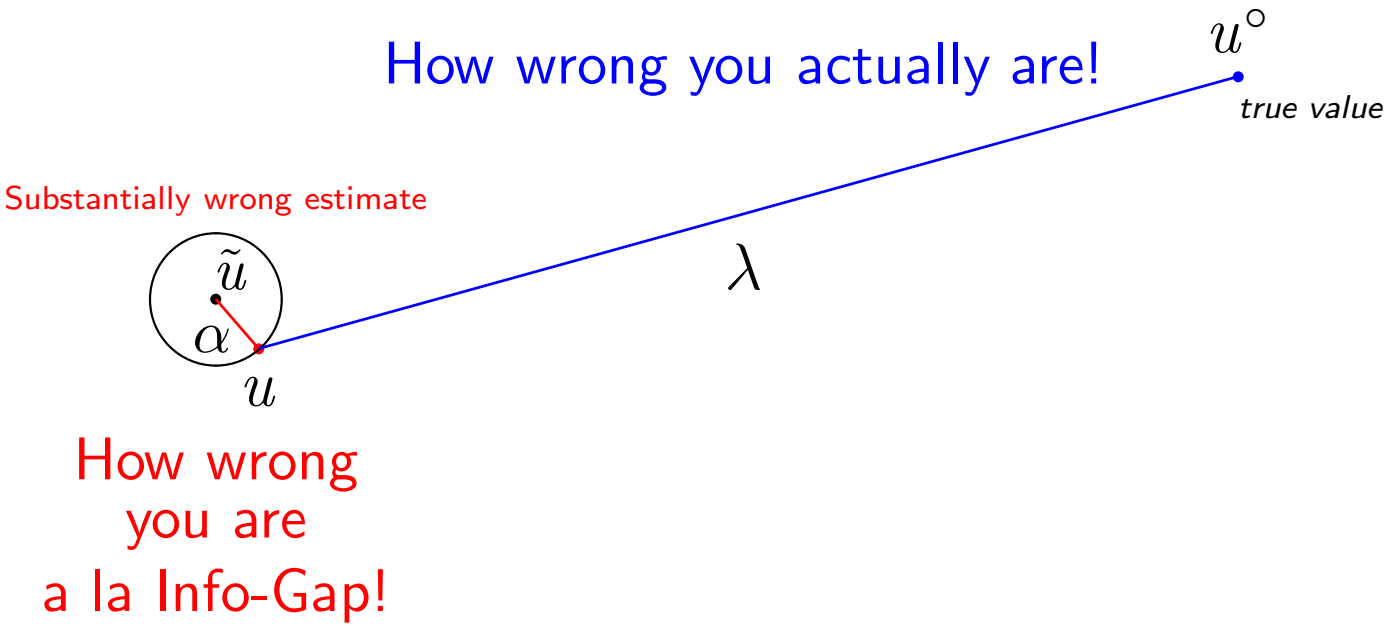
Fundamental Flaw



Myths and Facts

In short,

$$\text{True gap} = \lambda := ||u - u^\circ|| \ , \ u^\circ = \text{unknown true value}$$
$$\text{Info-Gap gap} = \alpha := ||u - \tilde{u}|| \ , \ \tilde{u} = \text{given poor estimate}$$



Who would be interested in the deviation from a substantially wrong estimate?

Myths and Facts

Myth # 6

Info-Gap generates robust solutions for decision-making problems under severe uncertainty.

Fact # 6

There is no reason to believe that this is so (see explanation and counter examples in Sniedovich [2006]).

Region of Severe Uncertainty, $\mathfrak{U}$

Voodooism

The Curse of Preference Reversal

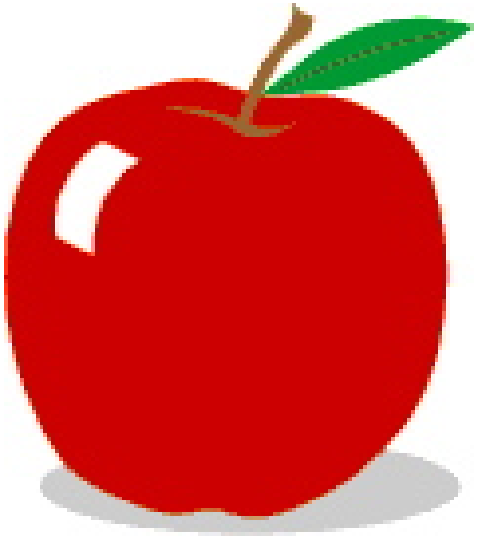
Region of Severe Uncertainty

Plan A is great!!
Plan B is a lemon!!

poor estimate

true value

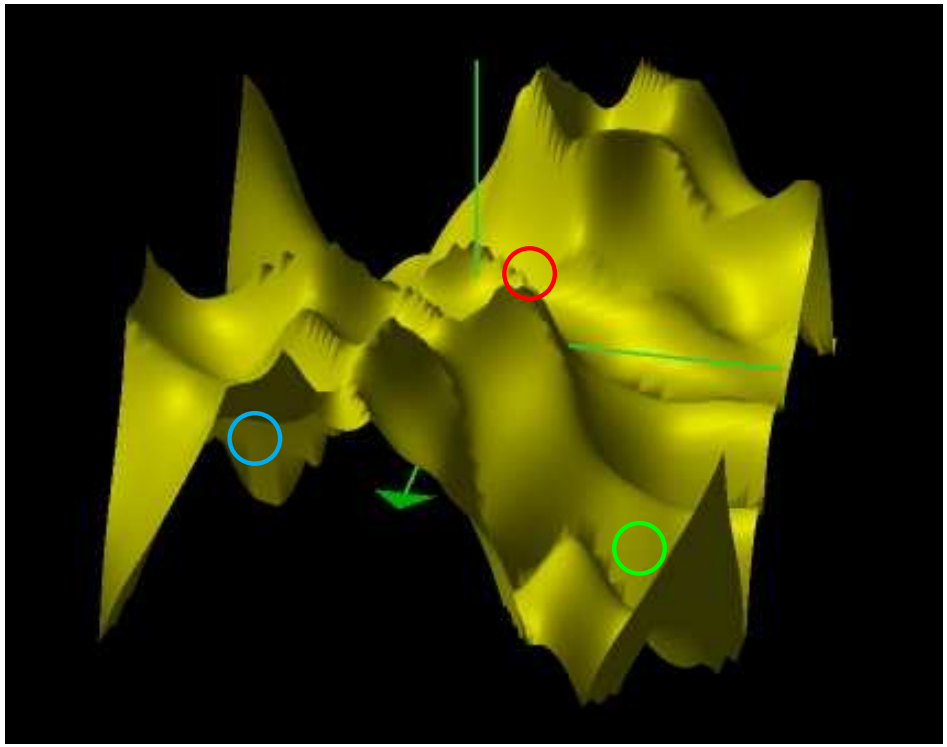
Plan A is a lemon!!
Plan B is great!!



VS

Myths and Facts

Note the analogy with the distinction between local and global optimization.

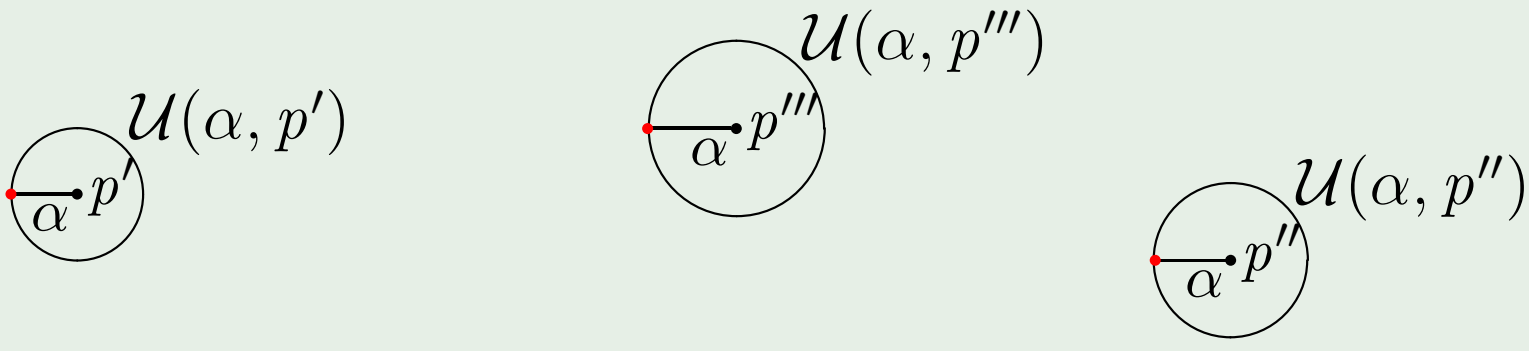


And . . . – of course – the analogy with the distinction between marginal and conditional probabilities.

Myths and Facts

Comment

To obtain a robust solution under severe uncertainty you have to incorporate in the analysis a number of point estimates, making sure that they adequately represent the entire region of uncertainty, $\mathfrak{U}$.



See the Worst-Case Analysis and Robust Optimization literature for tips, guidelines and inspiration.

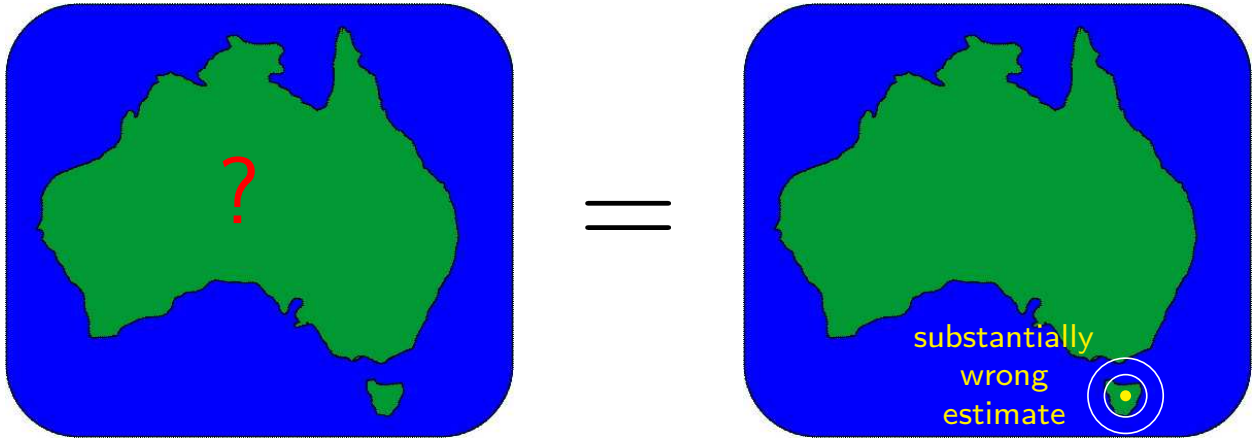
Conclusions

- Decision-making under severe uncertainty is difficult.
- It is a thriving area of research/practice.
- The Robust Optimization literature is extremely relevant.
- The Decision Theory literature is extremely relevant.
- The Operations Research literature is very relevant.
- Info-Gap's decision model is neither new nor radically different. It is a simple Maximin [1945] model.
- Info-Gap's uncertainty model is fundamentally flawed and unsuitable for decision-making under severe uncertainty.
- There is no reason to believe that Info-Gap generates robust decisions.

Conclusions

Breaking news from Voodooland . . .

Info-Gap is
a Voodoo decision-making theory
par excellence!



$$\alpha^* := \max_{q \in \mathbb{Q}} \; \max \left\{ \alpha \geq 0 : r_c \leq \min_{u \in \mathcal{U}(\alpha, \tilde{u})} R(q, u) \right\}$$

Bottom line

- Info-Gap is a Voodoo decision-making paradigm par excellence.
- It grossly misrepresents the state of the art in decision theory and related fields.
- It is time to reassess its role and place in decision theory and its use in Australia.

More ...

www.ms.unimelb.edu.au/∼moshe

Look for

- Info-Gap
- Maximin
- Voodoo decision-making

Join the Campaign!

The END

Myths and Facts

Info-Gap Quote of the Week
Great Expectations

Furthermore, we would like to be confident that this outcome will be achieved even if the models upon which our decision is based are substantially flawed.

p. 116

Planning for robust reserve networks using uncertainty analysis

Ecological Modelling, 2006, 115-124

Moilanen, Runge, Elith, Tyre, Carmel, Fegraus, Wintle, Burgman, Ben-Haim

This is in contradiction to the famous Fundamental Axiom:

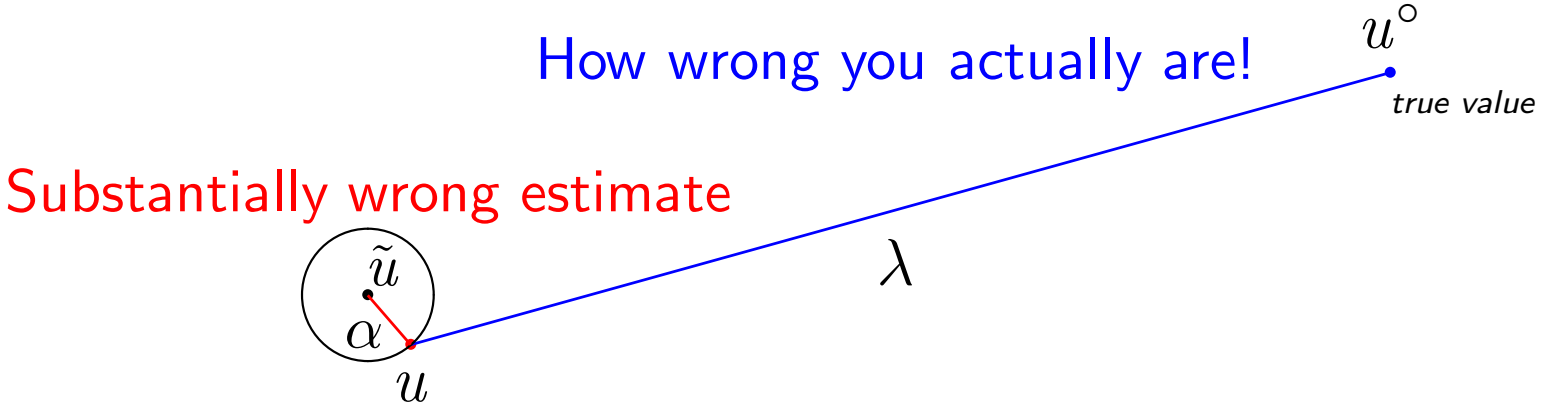
Wrong . . . in $\rightarrow$ **Model** $\rightarrow$ Wrong . . . out

Myths and Facts

Wrong . . . in → **Model** → Wrong . . . out

$$\text{True gap} = \lambda := ||u - u^{\circ}|| \tag{8}$$

$$\text{Info-Gap gap} = \alpha := ||u - \tilde{u}|| \tag{9}$$



Appendix

Myths and Facts

Myth # 7

Info-Gap's region of uncertainty is unbounded, therefore there is no worst case, and info-gap is not Maximin (Ben-Haim [2005]).

Info-Gap's claim is so outrageous, that . . . a quote might be needed for the unbelievers!

Myths and Facts

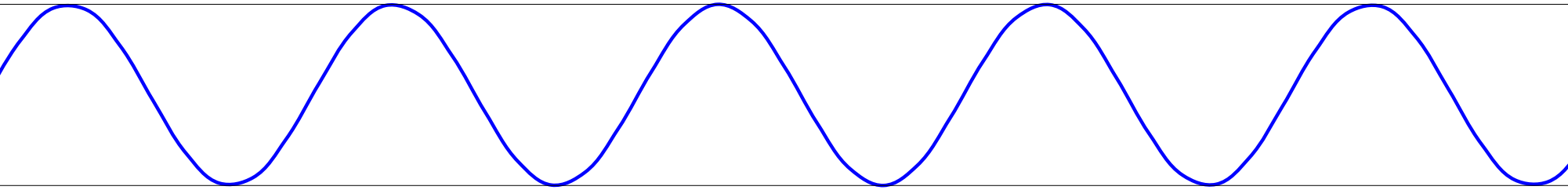
Ben-Haim (2005, p. 392)

It is important to emphasize that the robustness $\tilde{h}(R_*, c)$ is not a minimax algorithm. In minimax robustness analysis, one minimizes the maximum adversity. This is not what info-gap robustness does. There is no maximal adversity in an info-gap model of uncertainty: the worst case at any horizon of uncertainty h is less damaging than some realization at a greater horizon of uncertainty. Since the horizon of uncertainty is unbounded, there is no worst case and the info-gap analysis cannot and does not purport to ameliorate a worst case.

Myths and Facts

Fact # 9

- There is a worst case in all problems where Info-Gap yields a solution (Sniedovich [2006]).
- There can be a worst case even if the region of uncertainty is unbounded (eg $\sin(x), -\infty < x < \infty$).



Myths and Facts

620-161: Introductory Mathematics

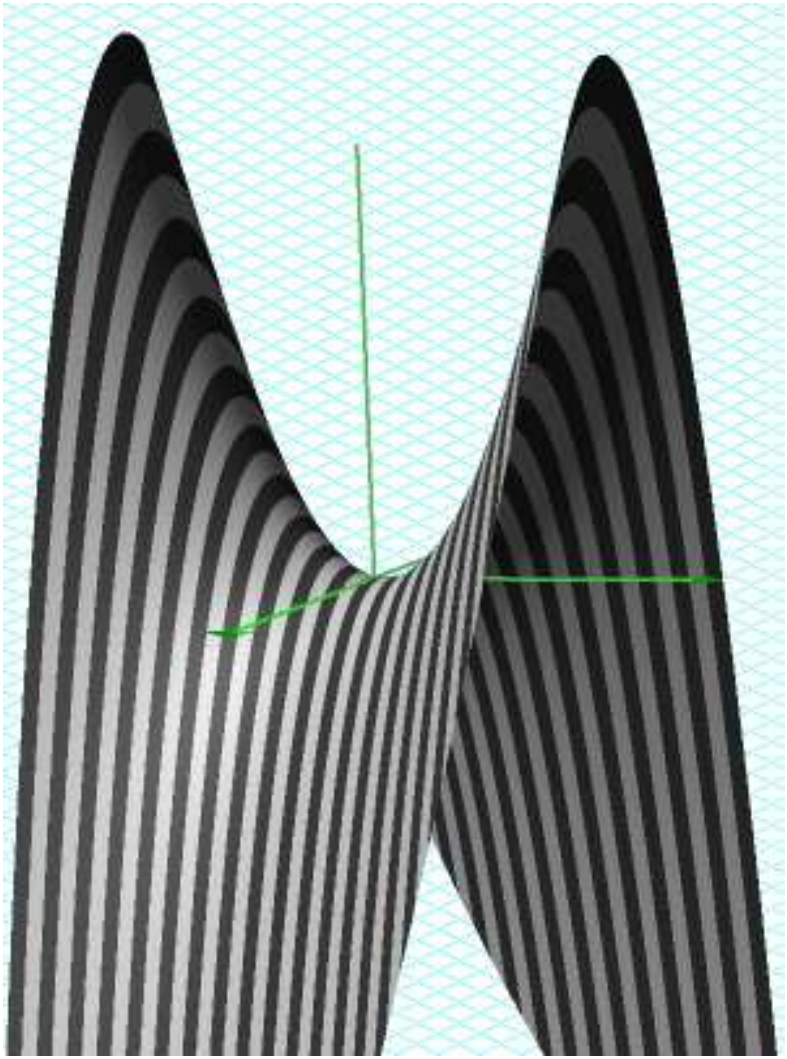
The most classical saddle point on Planet Earth is associated with the unbounded region $\mathbb{R}^2$ and the function

$$f(x, y) := x^2 - y^2$$

Its saddle point $(x, y) = (0, 0)$ is the solution to the Maximin problem

$$z^* := \max_{y \in \mathbb{R}} \min_{x \in \mathbb{R}} \left\{ x^2 - y^2 \right\}$$

$$z^* := \max_{y \in \mathbb{R}} \min_{x \in \mathbb{R}} \left\{ x^2 - y^2 \right\}$$



Myths and Facts

Ben-Haim [2001-2006] confuses various issues that are related to the structure of Info-Gap's uncertainty model:

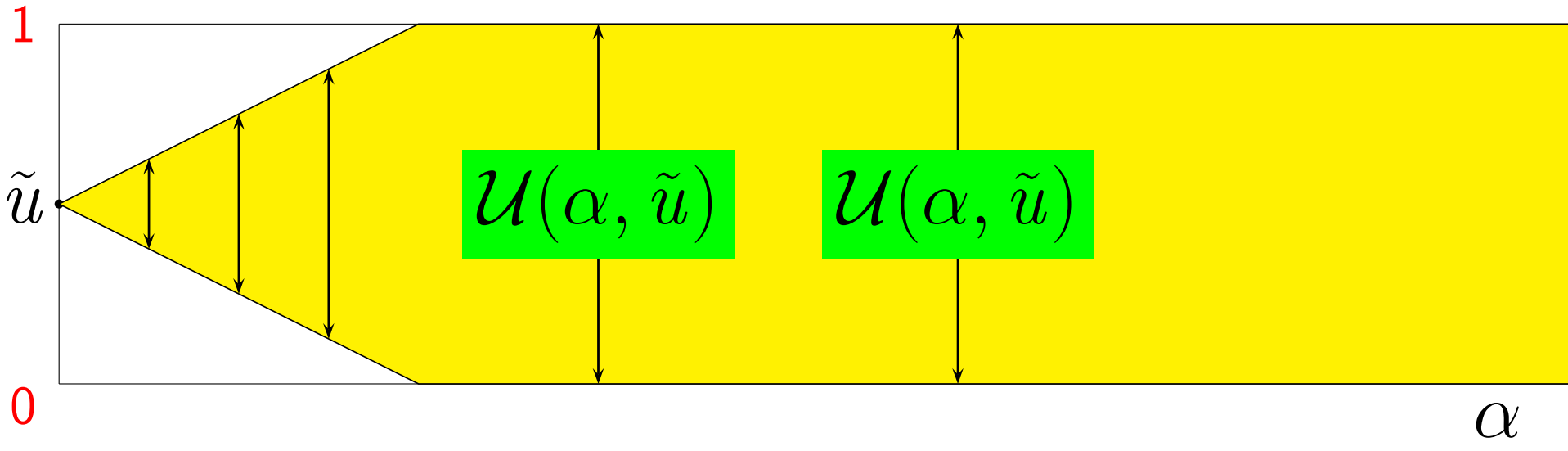
$$\alpha(r_c) := \max_{q \in \mathbb{Q}} \max \left\{ \alpha \geq 0 : r_c \leq \min_{u \in \mathcal{U}(\alpha, \tilde{u})} R(q, u) \right\}$$

- α is unbounded.
- $\mathcal{U}(\alpha, \tilde{u})$ is (???????) unbounded.
- $R(q, u)$ is (???????) unbounded.

Myths and Facts

Example (Ben-Haim [2006, pp. 256-7])

- $\mathcal{U}(\alpha, \tilde{u}) := \left\{ u \in [0,1] : \left| \frac{u - \tilde{u}}{\tilde{u}} \right| \leq \alpha \right\}$, $\alpha \geq 0$
- α is unbounded.
- $\mathcal{U}(\alpha, \tilde{u}) \subseteq [0,1]$ is bounded for all $\alpha \geq 0$.
- There is definitely a worst case!

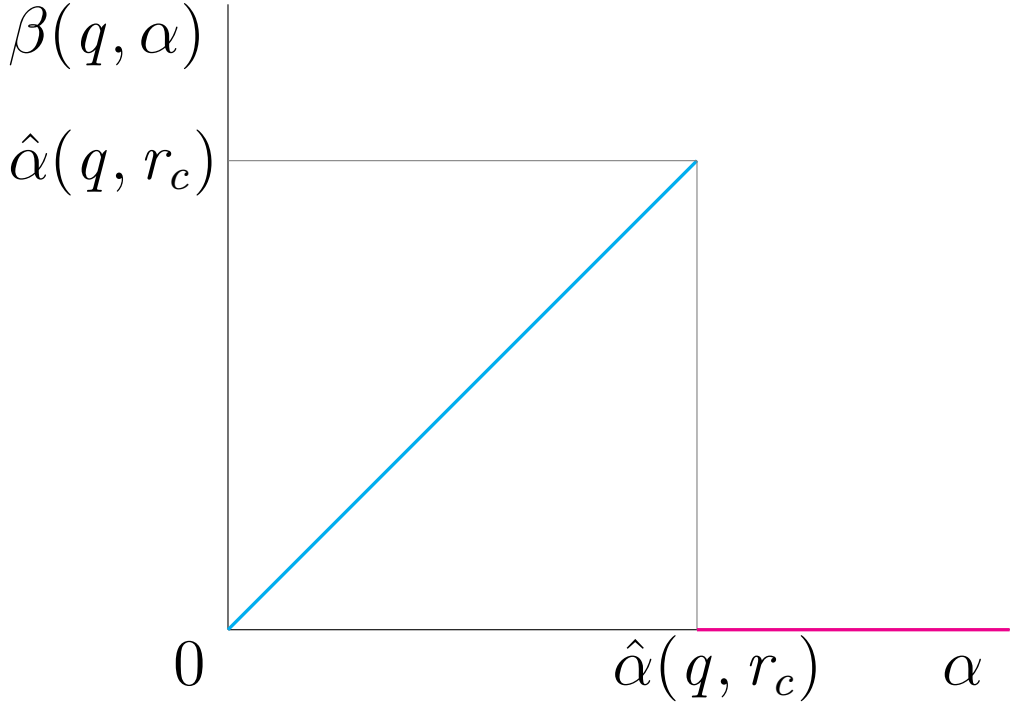


Myths and Facts

In the case of Info-Gap, the objective function is bounded:

$$\alpha(q, r_c) := \max_{\alpha \geq 0} \ \alpha \cdot \min_{u \in \mathcal{U}(\alpha, \tilde{u})} (r_c \preceq R(q, u))$$

$$\beta(q, r_c) := \alpha \cdot \min_{u \in \mathcal{U}(\alpha, \tilde{u})} (r_c \preceq R(q, u))$$



Bibliography

- Ben-Haim Y. Information Gap Decision Theory, Academic Press, 2001.
- Ben-Haim Y. Value-at-risk with info-gap uncertainty, *The Journal of Risk Finance*, 6(5), 388-403, 2005.
- Ben-Haim Y. Info-Gap Decision Theory, Elsevier, 2006.
- Ben-Tal A, El Ghaoui L, Nemirovski A. Mathematical Programming, Special issue on *Robust Optimization*, Volume 107(1-2), 2006.
- Carmel Y, Ben-Haim Y. Info-Gap Robust-Satisficing Model of Foraging Behavior: Do Foragers or Satisfice?, *The American Naturalist*, 166(5), 663-641, 2005.
- French SD. Decision Theory, Ellis Horwood, 1988.

Bibliography

- Grünig R, Kühn R. Successful Decision Making, Springer-Verlag, 2005.
- Kouvelis P, Yu G. Robust Discrete Optimization and Its Applications, Kluwer, 1997.
- Laskey, K.B.. and Lehner, P.E. Metareasoning and the Problem of Small Worlds, IEEE Transactions on Systems, Man, and Cybernetics, 24(11), 1643-1651, 1994.
- Rosenhead MJ, Elton M, Gupta SK. Robustness and Optimality as Criteria for Strategic Decisions, *Operational Research Quarterly*, 23(4), 413-430, 1972.

Bibliography

- Rustem B, Howe M. Algorithms for Worst-case Design and Applications to Risk Management, Princeton University Press, 2002.
- Sniedovich M. What's wrong with Info-Gap? An Operations Research Perspective, Working Paper MS-01-06, Department of Mathematics and Statistics, University of Melbourne, 2006.
- Tintner G. Abraham Wald's contributions to econometrics, *The Annals of Mathematical Statistics*, 23(1), 21-28, 1952.
- Vladimirou H, Zenios SA. Stochastic Programming and Robust Optimization, Chapter 12 in *Advances in Sensitivity Analysis and Parametric Programming*, edited by Gal T, Greenberg HJ, Kluwer, 1997.

Bibliography

- Wald A. Statistical decision functions which minimize the maximum risk, *The Annals of Mathematics*, 46(2), 265-280, 1945.
- Wald A. Statistical Decision Functions, John Wiley, 1950.
- Winston WL. Operations Research: Applications and Algorithms, Duxbury Press, 1994.