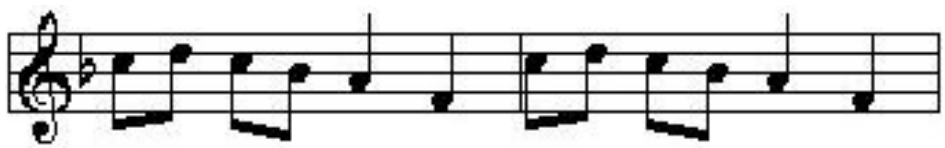
A Critique of Info-Gap Robustness Model

Moshe Sniedovich

Department of Mathematics and Statistics
The University of Melbourne
www.moshe-online.com



ESREL/SRA Conference
September 22-25, 2008
Valencia, Spain

Program

- Info-Gap decision theory – first encounter
- Classical decision theory
- Robust decision-making
- Voodoo decision theory
- Info-Gap decision theory – revisited

Admin

This is a

Math Classification G

presentation.

Math Classification MA +18

versions can be found at

`moshe-online.com`

Australian Perspective

New Secret Weapon Against Severe Uncertainty

$$\hat{\alpha}(q) := \max\{\alpha \geq 0 : r \leq R(q,u), \forall u \in U(\alpha, \tilde{u})\}, q \in \mathcal{Q}$$

Known as

Info-Gap Robustness Model

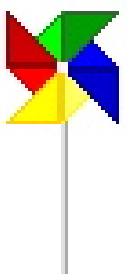
Ben-Haim (1996, 2001, 2006)

Very popular in a number of research organizations in Australia

This presentation

Objective of this presentation

- Overview of classical decision theory (1945-50)
- Overview of Robust Decision-Making
- Overview of Voodoo Decision-Making
- Discuss the role and place of Info-Gap Decision Theory in robust decision-making
- Report on my Maximin Campaign



Classification of Uncertainty

Classical decision theory distinguishes between three levels of uncertainty regarding the state of nature, namely

- Certainty
- Risk
- Strict Uncertainty

Terminology:

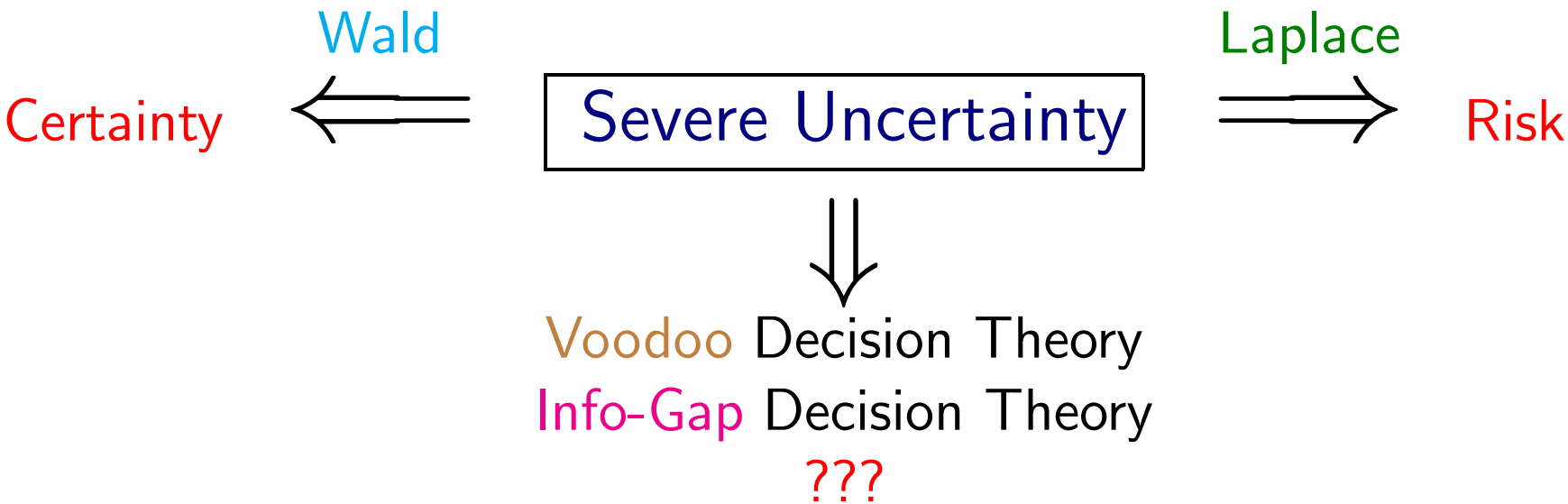
Strict Uncertainty ≡ Severe Uncertainty
≡ Ignorance
≡ True Uncertainty
≡ Knightian Uncertainty
≡ Deep
≡ Extreme
≡ Hard
≡ Fundamental

Severe Uncertainty

Classical decision theory offers two basic principles for dealing with severe uncertainty, namely

- Laplace's Principle (1825)
- Wald's Principle (1945)

Conceptually:



Laplace's Principle of Insufficient Reason (1825)

Assume that all the states are equally likely, thus use a uniform distribution function (μ) on the state space and regard the problem as decision-making under risk.

Laplace's Decision Rule

$$\max_{d\in\mathbb{D}} \int_{s\in S(d)} r(s,d)\mu(s)ds \qquad \text{Continuous case}$$

$$\max_{d\in\mathbb{D}} \frac{1}{|S(d)|} \sum_{s\in S(d)} r(s,d) \qquad \text{Discrete case}$$

Wald's Maximin Principle (1945)

Inspired by Von Neumann's [1928] Maximin model for 0-sum, 2-person games: Mother Nature is and adversary and is playing against you, hence apply the worst-case scenario. This transforms the problem into a decision-making under certainty.

Nice Plain Language Formulation

The maximin rule tells us to rank alternatives by their worst possible outcomes: we are to adopt the alternative the worst outcome of which is superior to the worst outcome of the others.

Rawls, J., *Theory of Justice*, 1971, p. 152

Wald's Maximin Principle (1945)

Historical perspective

The gods to-day stand friendly, that we may,
Lovers of peace, lead on our days to age!
But, since the affairs of men rests still incertain,
Let's reason with the worst that may befall.

William Shakespeare (1564-1616)
Julius Caesar, Act 5, Scene 1

Classic Format

$$\max_{d \in \mathbb{D}} \quad \min_{s \in S(d)} \quad f(d, s)$$

(You! Mama)

About Maximin/Minimax formulations

Classical Format

$$\overset{\text{You!}}{\max_{d\in\mathbb{D}}} \quad \overset{\text{Mama}}{\min_{s\in S(d)}} \quad f(d,s)$$

Mathematical Programming Format

$$\overset{\text{You!}}{\max_{\substack{d\in\mathbb{D}\\ v\in\mathbb{R}}}} \left\{ v : f(d,s) \geq v \ , \ \overset{\downarrow\ \text{Mama}}{\forall s \in S(d)} \right\}$$

Note: if $S(d)$ is "continuous", then this is a semi-infinite program.

Robust Decision-Making

WIKIPEDIA

Robustness is the quality of being able to withstand stresses, pressures, or changes in procedure or circumstance. A system, organism or design may be said to be "robust" if it is capable of coping well with variations (sometimes unpredictable variations) in its operating environment with minimal damage, alteration or loss of functionality.

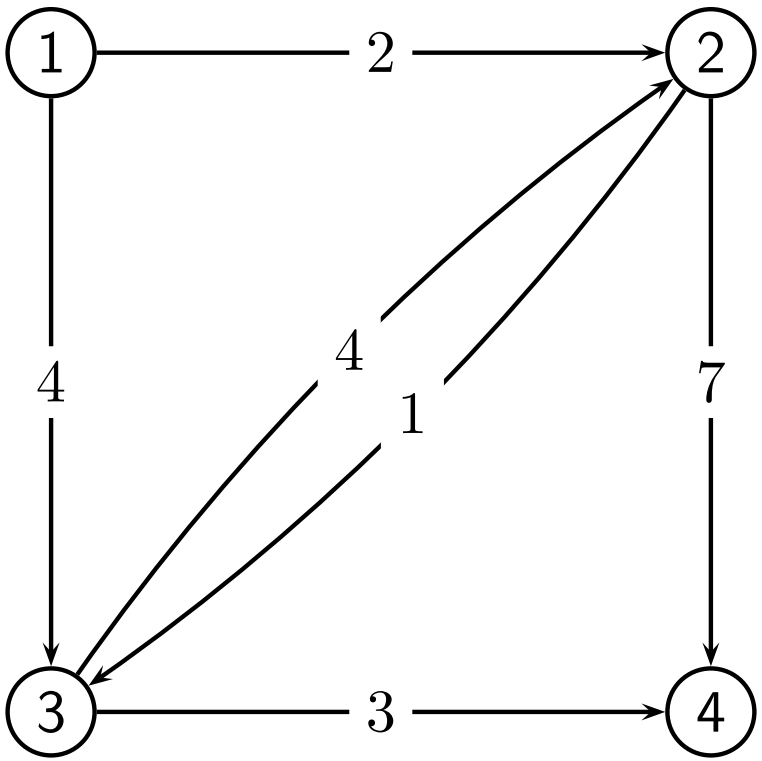
- Applies to both (known) variability and uncertainty
- Origin: probably late 1920's (game theory).
- In OR and Optimization: late 1960s early 1970s.
- Major difficulty: solution procedures.
- A very "hot" area of research these days ...
- See bibliography

Robust Optimization

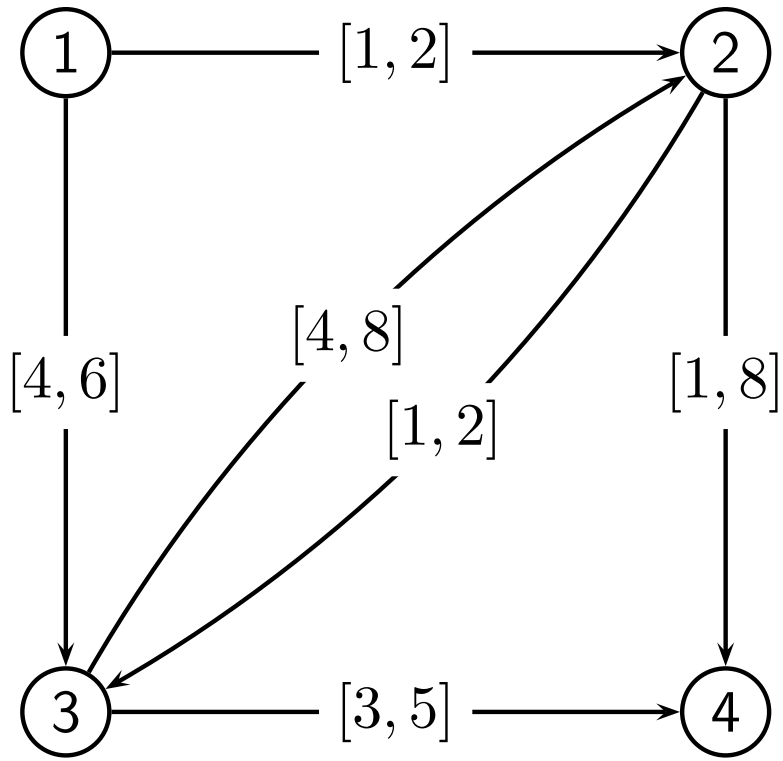
Simple Example

Shortest path problem with variable arc lengths

"Conventional version"



"Robust version"



Robust Decision-Making

Role of Maximin/Minimax in Robustness Analysis

But as we defined robustness to mean insensitivity with regard to small deviations from assumptions, any quantitative measure of robustness must somehow be concerned with the maximum degradation of performance possible for an ϵ-deviation from the assumptions. The optimally robust procedure minimizes this degradation and hence will be a minimax procedure of some kind.

Huber (1981, pp. 16-17)

Experience: Modeling aspects can be subtle!

- Optimizing vs Satisficing
- Complete vs Partial vs Local
- (Mis) Interpretation

Voodoo Decision Theory

Voodoo Decision Theory

Encarta online Encyclopedia

Voodoo n

1. A religion practiced throughout Caribbean countries, especially Haiti, that is a combination of Roman Catholic rituals and animistic beliefs of Dahomean enslaved laborers, involving magic communication with ancestors.
2. Somebody who practices voodoo.
3. A charm, spell, or fetish regarded by those who practice voodoo as having magical powers.
4. A belief, theory, or method that lacks sufficient evidence or proof.

Voodoo Decision Theory

Apparently very popular,

Example

The behavior of Kropotkin's cooperators is something like that of decision makers using Jeffrey expected utility model in the Max and Moritz situation. Are ground squirrels and vampires using voodoo decision theory?

Brian Skyrms
Evolution of the Social Contract
Cambridge University Press, 1996.

Issue:

Evidential dependence, but causal independence.

The legend

An old legend has it that an ancient treasure is hidden in an Asian-Pacific island.

You are in charge of the treasure hunt. How would you plan the operation?

The legend

Main issue: location, location, location!

Terminology

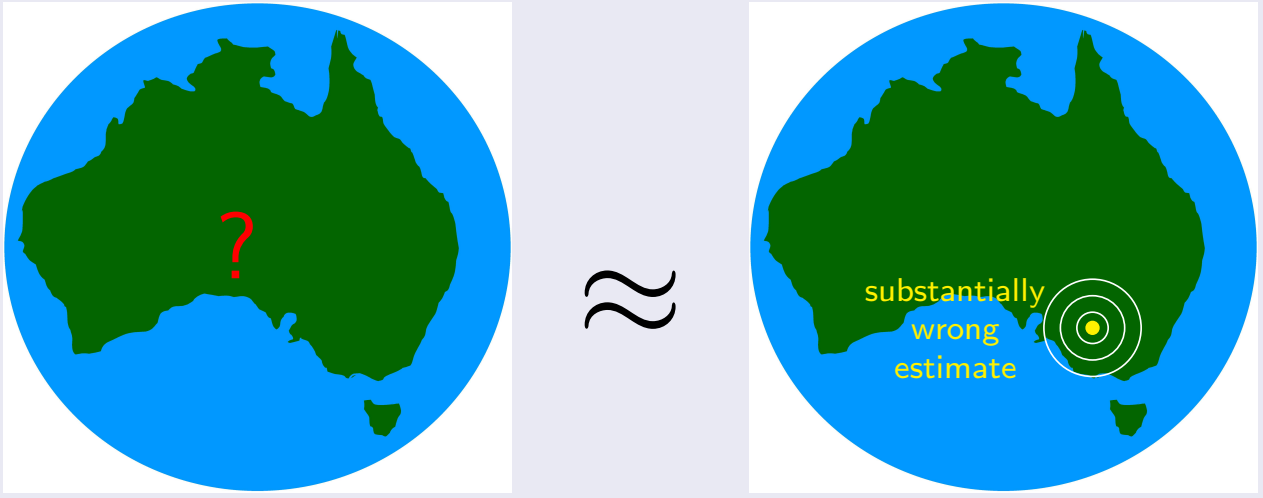
Certainty

Risk

Severe Uncertainty

Voodooism

The Fundamental Theorem of Voodoo Decision Making



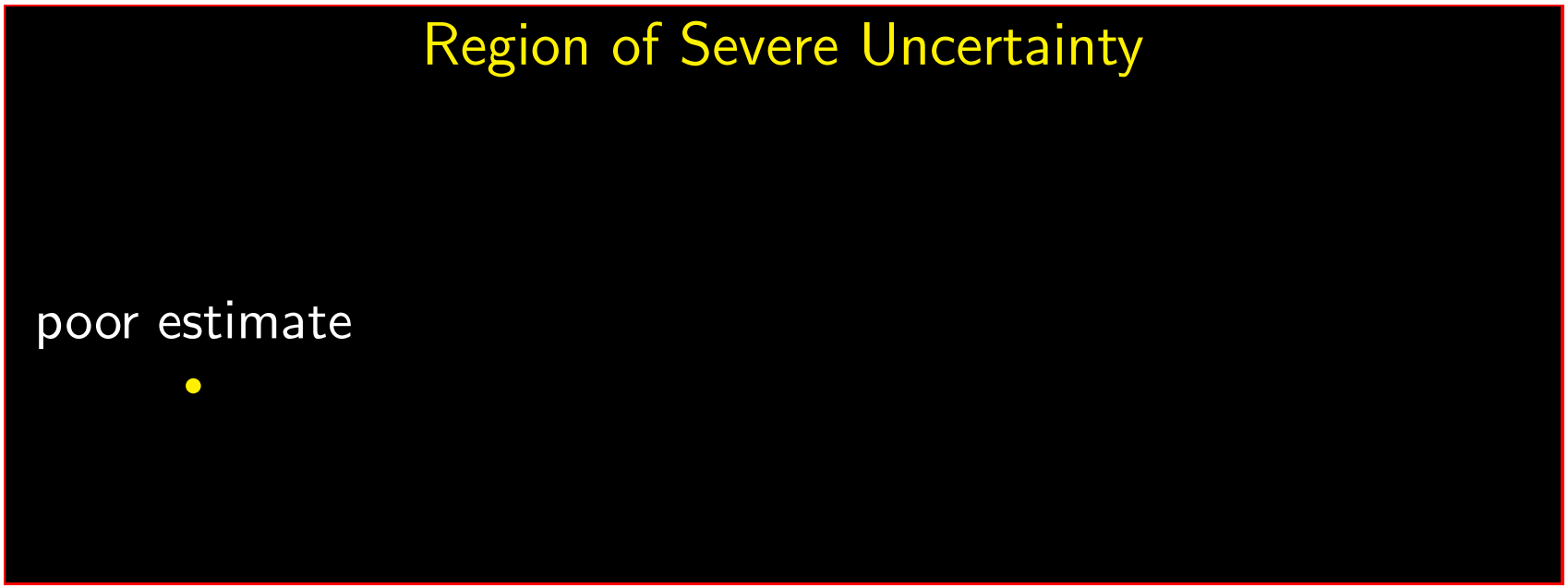
Severe Uncertainty

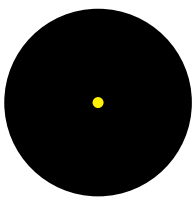
1·2·3 Recipe

1. Ignore the severe uncertainty.
2. Focus on the substantially wrong estimate you have.
3. Conduct the analysis in the immediate neighborhood of this estimate.

Voodooism

Voodoo Decision-Making





Voodooism

Conventional Decision Theory

$$\text{GI} \longrightarrow \boxed{\text{Model}} \longrightarrow \text{GO}$$

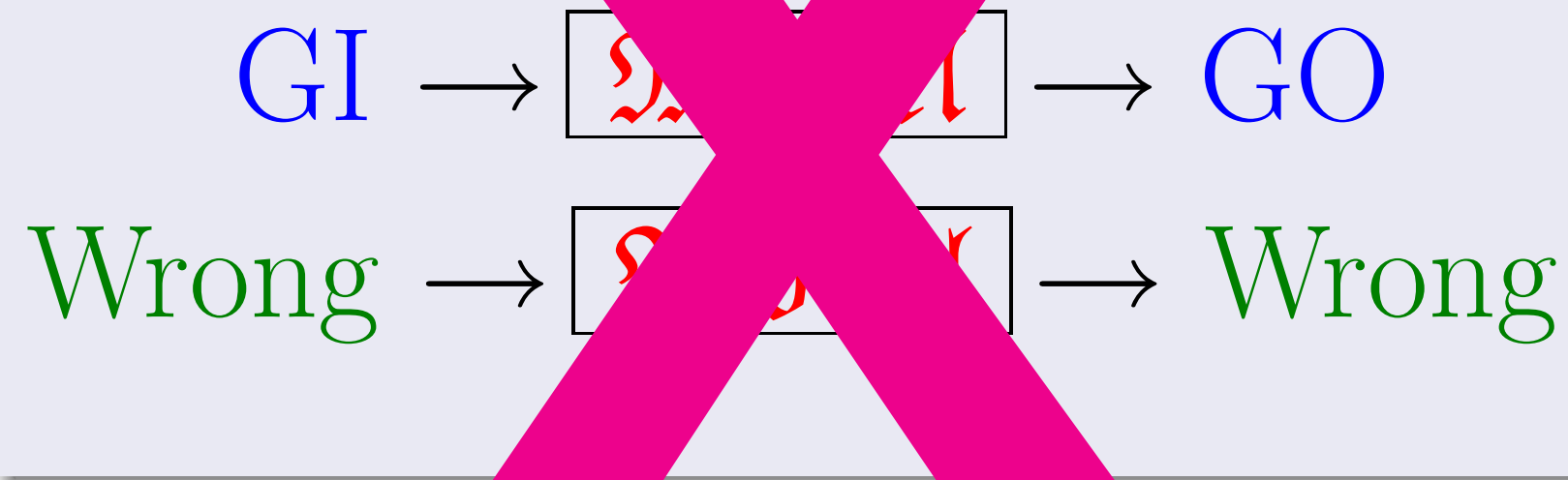
$$\text{Wrong} \longrightarrow \boxed{\text{Model}} \longrightarrow \text{Wrong}$$

The robustness of any decision and the risk incurred in making that decision is only as good as the estimates on which it is based. Making estimation even more challenging, virtually all estimates that affect decisions are uncertain. Uncertainty can not be eliminated, but it can be managed.

Top Ten Challenges for Making Robust Decisions
The Decision Expert Newsletter, Volume 1; Issue 2
http://www.robustdecisions.com/newsletter0102.php

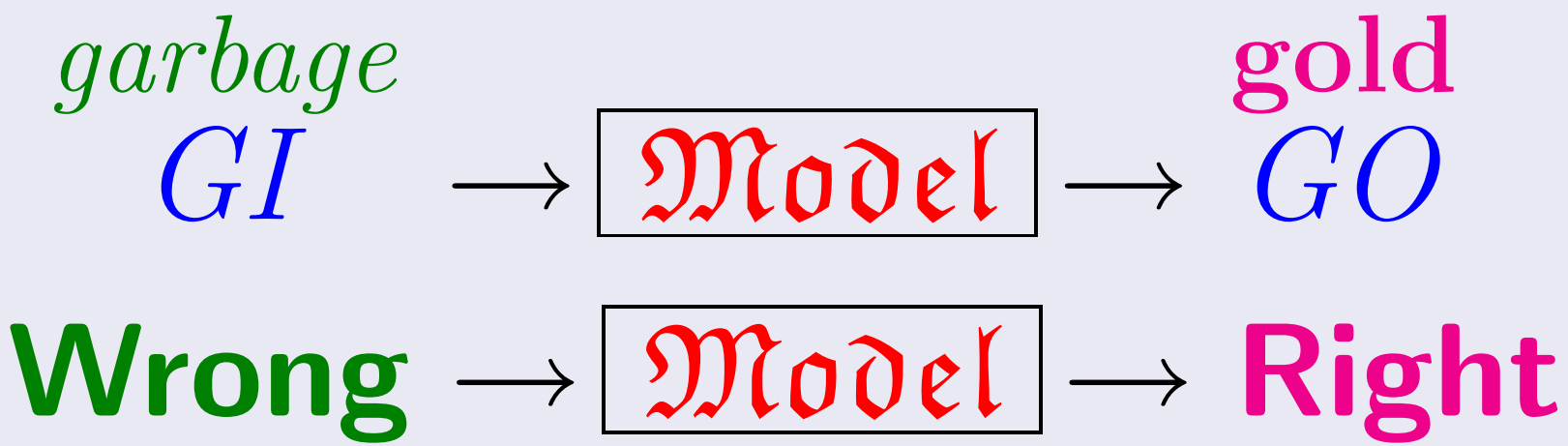
Voodooism

V Decisio ory



Voodooism

Voodoo Decision Theory



Alchemy

Info-Gap Revisited

Impressive Self-Portrait

Info-gap decision theory is radically different from all current theories of decision under uncertainty. The difference originates in the modelling of uncertainty as an information gap rather than as a probability. The need for info-gap modeling and management of uncertainty arises in dealing with severe lack of information and highly unstructured uncertainty.

Ben-Haim [2006, p. xii]

In this book we concentrate on the fairly new concept of information-gap uncertainty, whose differences from more classical approaches to uncertainty are real and deep.

Ben-Haim [2006, p. 11]

Info-Gap

Obvious Questions

1. Does Info-Gap substantiate these very strong claims?
2. Are these claims valid?

Not So Obvious Answers

1. No, it does not.
2. Certainly not.

It is therefore important to subject Info-Gap to a formal analysis – that actually should have been done seven years ago:

Info-Gap

Formal vs Analysis

Classical Decision Theory

Good news: should take no more than 5-10 minutes!

Info-Gap

Meaning of Severe Uncertainty

- The region of uncertainty is usually relatively large, often unbounded.
- The uncertainty cannot be quantified by a probabilistic model.
- If there is an estimate of the parameter of interest, then the estimate is
  - A wild guess
  - A poor indication of the true value
  - Likely to be substantially wrong

Info-Gap

Practical Meaning of Severe Uncertainty

bio-security homeland-security

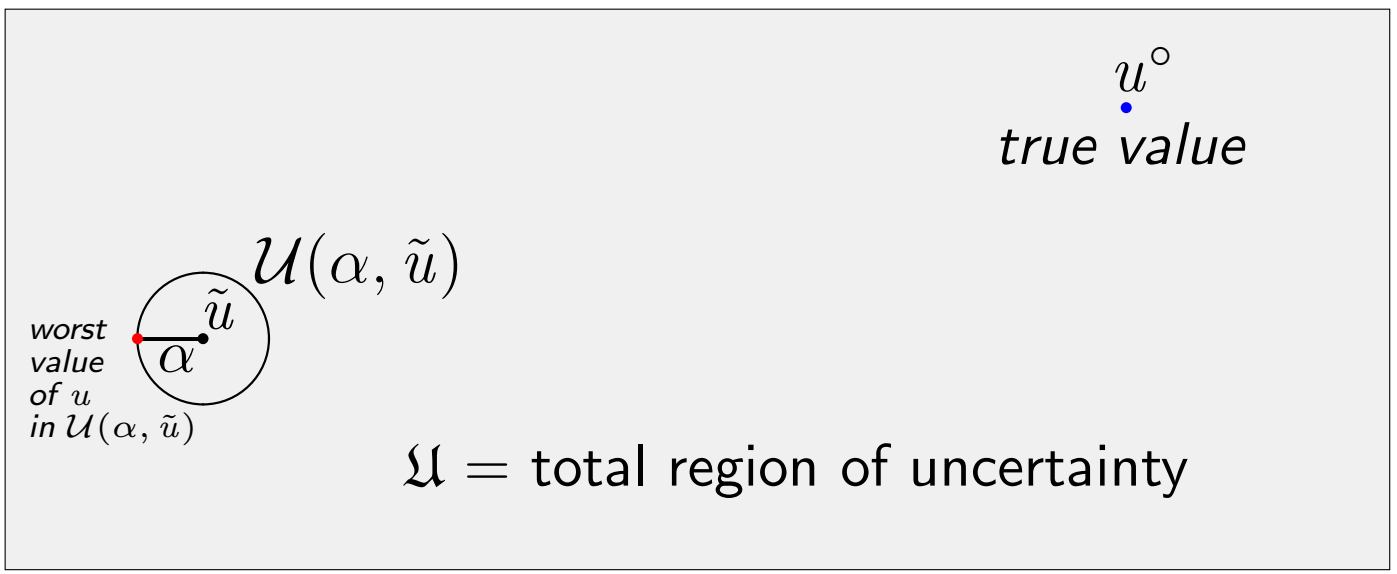
Info-Gap Decision Theory

Complete Generic Robustness Model

$$\hat{\alpha}(r_c) := \max_{q \in \mathbb{Q}} \max \left\{ \alpha \geq 0 : r_c \leq \min_{u \in \mathcal{U}(\alpha, \tilde{u})} R(q, u) \right\}$$

$$\mathcal{U}(\alpha, \tilde{u}) \subseteq \mathcal{U}(\alpha + \varepsilon, \tilde{u}), \forall \varepsilon > 0$$

Region of Severe Uncertainty, $\mathfrak{U}$



Info-Gap Decision Theory

Complete Generic Robustness Model

$$\hat{\alpha}(r_c) := \max_{q \in \mathbb{Q}} \max \left\{ \alpha \geq 0 : r_c \leq \min_{u \in \mathcal{U}(\alpha, \tilde{u})} R(q, u) \right\}$$

Fundamental FAQs

1. Is this new? Definitely not!
2. Is this radically different? Definitely not!
3. Does it make sense? Definitely not!

So what is all this hype about Info-Gap ?!

Good question!

Info-Gap Decision Theory

First Impression

Complete Generic Robustness Model

$$\hat{\alpha}(r_c) := \max_{q \in \mathbb{Q}} \max \left\{ \alpha \geq 0 : r_c \leq \min_{u \in \mathcal{U}(\alpha, \tilde{u})} R(q, u) \right\}$$

Observations

- This model does not deal with severe uncertainty, it simply and unceremoniously ignores it.
- The analysis is invariant with $\mathfrak{U}$: the same solution for all $\mathfrak{U}$ such that $\mathcal{U}(\hat{\alpha}(r_c), \tilde{u}) \subseteq \mathfrak{U}$.
- This model is fundamentally flawed.
- This model advocates voodoo decision-making.

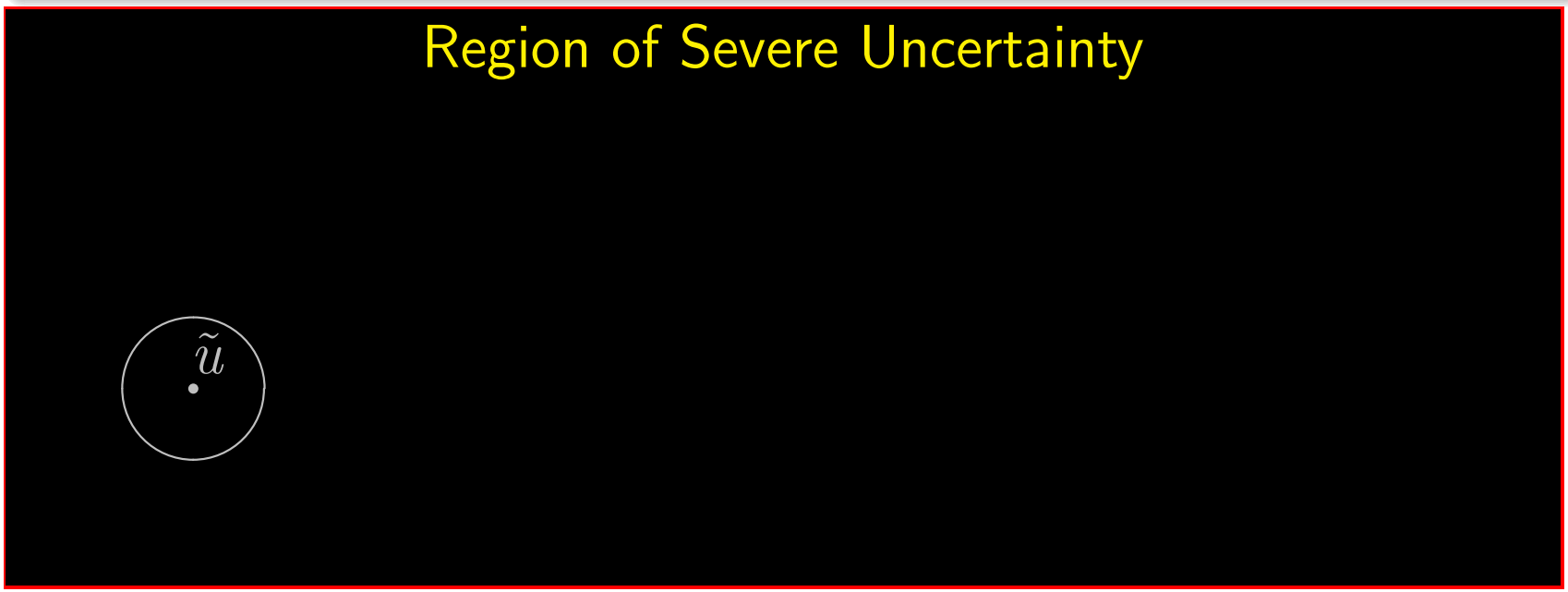
Info-Gap Decision Theory

First Impression

Fool-Proof Recipe

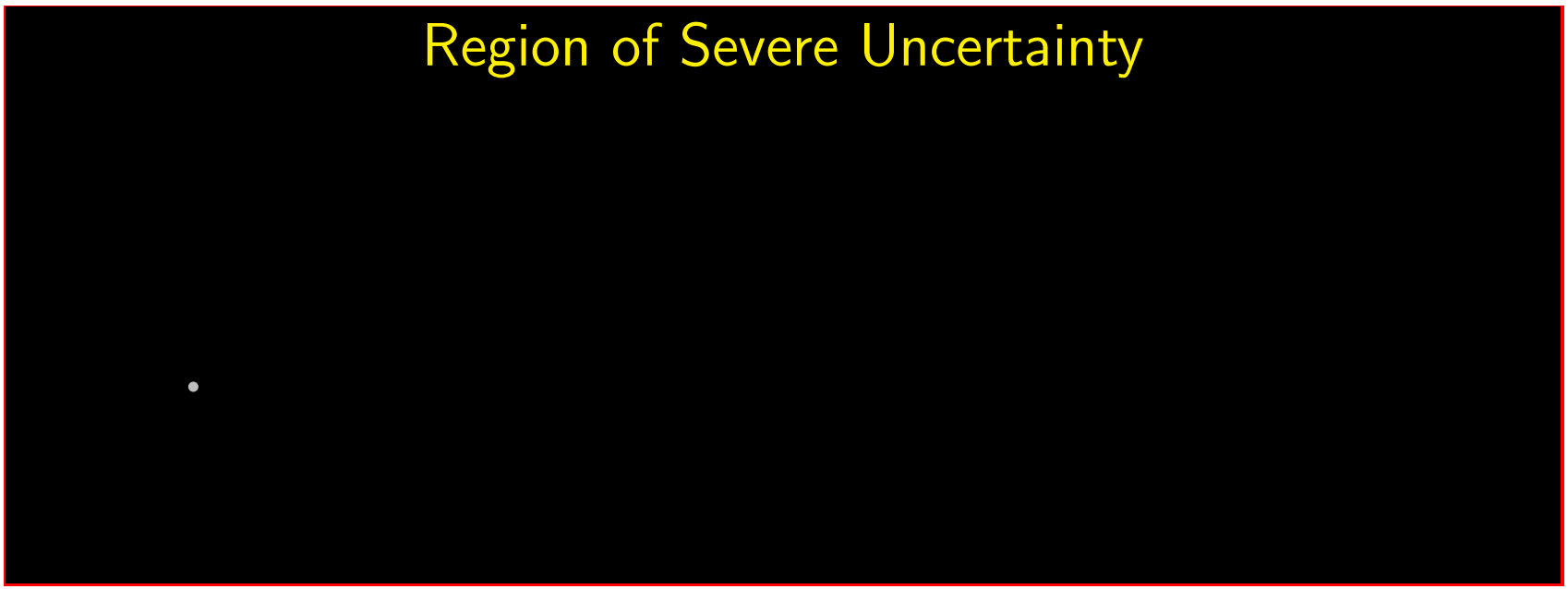
Step 1: *Ignore the severe uncertainty.*

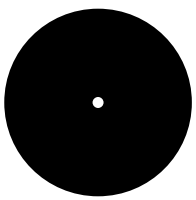
Step 2: *Focus instead on the poor estimate and its immediate neighborhood.*



Info-Gap Decision Theory

First Impression





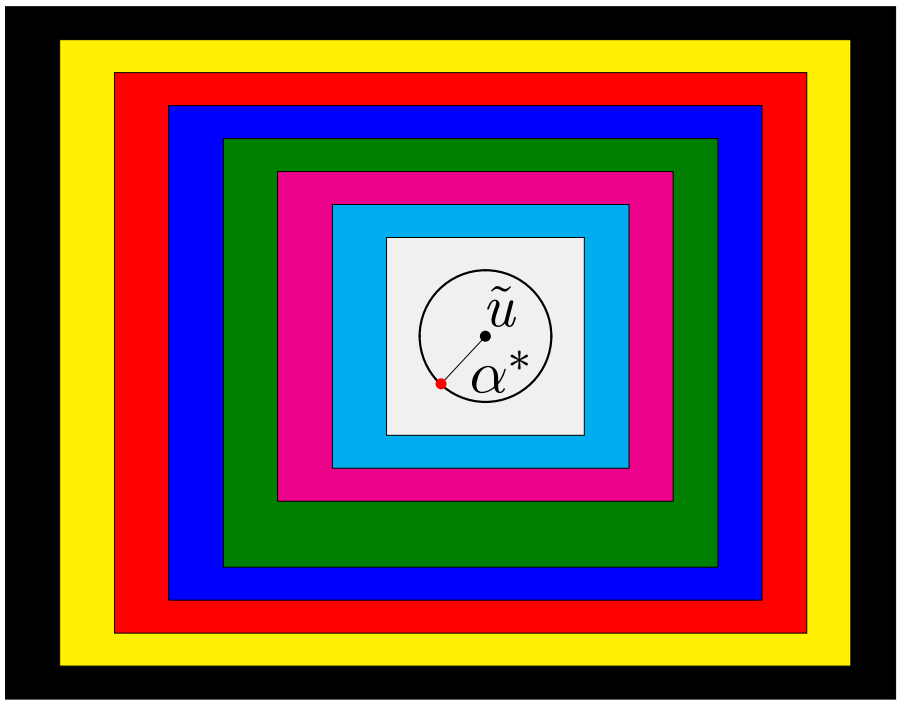
Recall that this is voodoo decision making!

Info-Gap Decision Theory

Complete Generic Robustness Model

$$\alpha^* := \max_{q \in \mathbb{Q}} \max \left\{ \alpha \geq 0 : r_c \leq \min_{u \in \mathcal{U}(\alpha, \tilde{u})} R(q, u) \right\}$$

Fundamental Flaw



Info-Gap Decision Theory

More formally

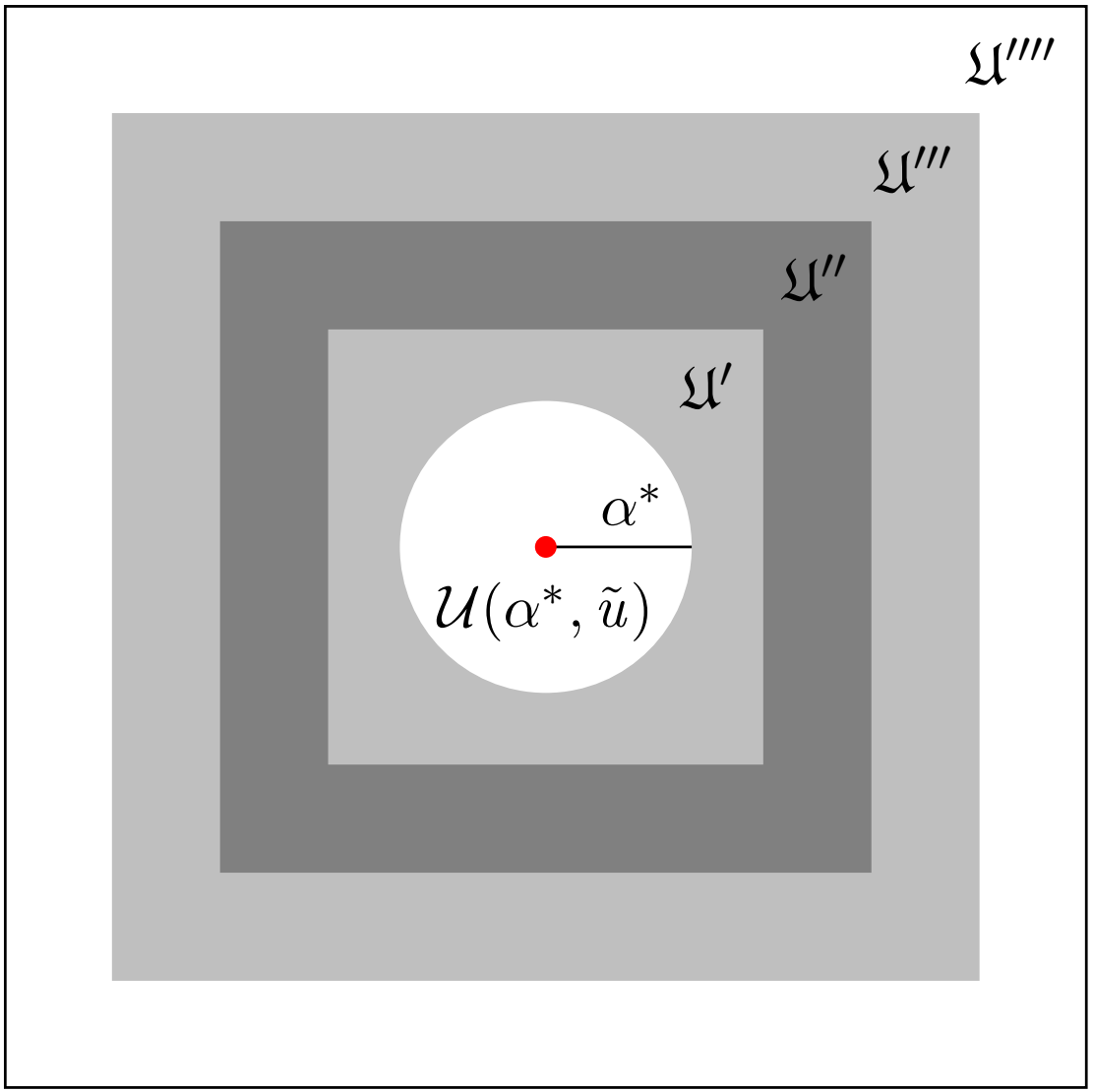
Invariance Theorem (Sniedovich, 2007)

Info-Gap's robustness model is invariant to the size of the total region of uncertainty $\mathfrak{U}$ for all $\mathfrak{U}$ larger than $\mathcal{U}(\alpha^*, \tilde{u})$, where $\alpha^* := \hat{\alpha}(r_c)$.
That is, the model yields the same results for all $\mathfrak{U}$ such that

$$\mathcal{U}(\alpha^* + \varepsilon, \tilde{u}) \subseteq \mathfrak{U} \ , \ \varepsilon > o$$

Info-Gap Decision Theory

Info-Gap's Invariance Property



Info-Gap Decision Theory

Maximin Theorem (Sniedovich 2007, 2008)

Info-Gap's robustness model is a simple instance of Wald's Maximin model. Specifically,

$$\begin{aligned}\alpha(q) :&= \max_{\alpha \geq 0} \left\{ \alpha : r_c \leq \min_{u \in \mathcal{U}(\alpha, \tilde{u})} R(q,u) \right\} \ , \ q \in \mathbb{Q} \\ &= \max_{\alpha \geq 0} \min_{u \in \mathcal{U}(\alpha, \tilde{u})} \psi(q, \alpha, u)\end{aligned}$$

where

$$\psi(q,\alpha,u) := \begin{cases} \alpha \ , & r_c \leq R(q,u) \\ 0 \ , & r_c > R(q,u) \end{cases} \ , \alpha \geq 0, q \in \mathbb{Q}, u \in \mathcal{U}(\alpha, \tilde{u})$$

Conclusions

- Decision-making under severe uncertainty is difficult.
- It is a thriving area of research/practice.
- The Robust Optimization literature is extremely relevant.
- The Decision Theory literature is extremely relevant.
- The Operations Research literature is very relevant.
- Info-Gap's robustness model is neither new nor radically different.
- Info-Gap's uncertainty model is fundamentally flawed and is unsuitable for decision-making under severe uncertainty.
- Info-Gap Decision Theory exhibits a severe information-gap about the state of the art in decision-making under severe uncertainty.

FAQs?

Bibliography

- Ben-Haim, Y. 1996. *Robust Reliability in the Mechanical Science,* Springer Verlag.
- Ben-Haim, Y. (1999). Design certification with information-gap uncertainty, *Structural Safety,* 2, 269-289.
- Ben-Haim, Y. 2001. *Information Gap Decision Theory.* Academic Press.
- Ben-Haim, Y. 2006. *Info-Gap Decision Theory.* Elsevier.
- Ben-Haim, Y. 2008. Info-Gap Forecasting and the Advantage of Sub-Optimal Models, European Journal of Operational Research, in press.
- Ben-Tal A. & Nemirovski, A. 1999. Robust solutions of uncertain linear programs, *OR Letters*, 25, 1-13.

- Ben-Tal A. & Nemirovski, A. 2002. Robust optimization – methodology and applications, *Mathematical Programming, Ser. B,* 92, 453?480.
- Ben-Tal A. El Ghaoui, L. & Nemirovski, A. 2006. *Mathematical Programming*, Special issue on *Robust Optimization,* 107(1-2).
- Cagetti, M., Hansen, L.P., Sargent, T., and Williams, N. 2002. Robustness and Pricing with Uncertain Growth, *The Review of Financial Studies,* 15, 2, 363-404.
- Davidovitch, L. and Ben-Haim, Y. 2008. *Profiling for crime reduction under severely uncertain elasticities*, working paper www.technion.ac.il/ yakov/IGT/lior15prof.pdf.
- Dembo, R.S. 1991. Scenario optimization. *Annals of Operations Research* 30(1): 63-80.

- Demyanov, V.M. and Malozemov, V.N. 1990. *Introduction to Minimax,* Dover.
- Du, D.Z. and Pardalos, P.M. 1995. *Minimax and Applications*, Springer Verlag.
- Eiselt, H.A., Sandblom, C.L. and Jain, N. 1998. A Spatial Criterion as Decision Aid for Capital Projects: Locating a Sewage Treatment Plant in Halifax, Nova Scotia, *Journal of the Operational Research Society,* 49(1), 23-27.
- Eiselt, H.A. and Langley A. 1990. Some extensions of domain criteria in decision making under uncertainty, *Decision Sciences,* 21, 138-153.
- Francis, R.L., McGinnis, Jr, L.F. & White, J.A. 1992. *Facility Layout and Location: An Analytical Approach.* Prentice Hall.

- French, S.D. 1988. *Decision Theory*, Ellis Horwood.
- Hall, J. & Ben-Haim, Y. 2007. Making Responsible Decisions (When it Seems that You Can't). www.floodrisknet.org.uk/a/2007/11/hall -benhaim.pdf.
- Huber, P.J. 1981. Robust Statistics. Wiley, New York.
- Kouvelis, P. & Yu, G. 1997. *Robust Discrete Optimization and Its Applications.*, Kluwer.
- Rawls, J. (2005). *Theory of Justice,* Belknap Press, Cambridge, MA.
- Reemstem, R. and Rückmann, J. (1998). *Semi-Infinite Programming,* Kluwer, Boston.
- Resnik, M.D. 1987. *Choices: an Introduction to Decision Theory.* University of Minnesota Press: Minneapolis.

- Rosenhead M.J, Elton M, Gupta S.K. 1972. Robustness and Optimality as Criteria for Strategic Decisions, *Operational Research Quarterly*, 23(4), 413-430.
- Rustem, B. & Howe, M. 2002. *Algorithms for Worst-case Design and Applications to Risk Management.* Princeton University Press.
- Schneller G.O. and Sphicas, G.P. (1983). Decision making under uncertainty: Starr's Domain criterion, Theory and Decision, 15, 321-336.
- Skyrms, B. 1996. *Evolution of the Social Contract,* Cambridge University Press.
- Sniedovich, M. 2007. The art and science of modeling decision-making under severe uncertainty. *Journal of Manufacturing and Services,* 1(1-2): 111-136.

- Sniedovich, M. 2008. Wald's Maximin Model: A Treasure in Disguise! *Journal of Risk Finance,* 9(3), 287-291.
- A. L. Soyster, A.L. 1973. Convex Programming with Set-Inclusive Constraints and Applications to Inexact Linear Programming, it Operations Research, 21(5), 1154-1157.
- Starr, M.K. 1962. *Product design and decision theory,* Prentice-Hall, Englewood Cliffs, NJ.
- Starr, M. K. 1966. A Discussion of Some Normative Criteria for Decision-Making Under Uncertainty, *Industrial Management Review,* 8(1), 71-78.
- Tintner, G. 1952. Abraham Wald's contributions to econometrics. *The Annals of Mathematical Statistics* 23(1): 21-28.

- Vladimirou, H. & Zenios, S.A. 1997. Stochastic Programming and Robust Optimization. In Gal, T, & Greenberg H.J. (ed.), *Advances in Sensitivity Analysis and Parametric Programming*. Kluwer.
- von Neumann, J. 1928. Zur theories der gesellschaftsspiele, *Math. Annalen*, Volume 100, 295-320.
- von Neumann, J. and Morgenstern, O. 1944. *Theory of Games and Economic Behavior*, Princeton University Press.
- Wald, A. 1945. Statistical decision functions which minimize the maximum risk, *The Annals of Mathematics,* 46(2), 265-280.
- Wald, A. 1950. *Statistical Decision Functions.*John Wiley.