Responsible Decision-Making in the Face of Severe Uncertainty

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Abstract & Program

- How do you make responsible decisions in the face of severe uncertainty?
  - Australian perspective
  - Info-Gap decision theory
  - Classical decision theory
  - Robust decision-making
  - Voodoo decision theory
  - Australian perspective – revised
    - My Maximin (and related) "Campaigns"
    - Info-gap decision theory – revisited
  - FAQs

Admin

This is a

Math Classification G

presentation.

Math Classification MA +18

versions can be found at

moshe-online.com

AU Perspective: Example

Planning for robust reserve networks using uncertainty analysis

. . . In summary, we recommend info-gap uncertainty analysis as a **standard practice** in computational reserve planning. The need for robust reserve plans may change the way biological data are interpreted. It also may change the way reserve selection results are evaluated, interpreted and communicated. Information-gap decision theory provides a standardized methodological framework in which implementing reserve selection uncertainty analyses is relatively straightforward. We believe that alternative planning methods that consider robustness to model and data error should be preferred whenever models are based on uncertain data, which is probably the case with nearly all data sets used in reserve planning . . .

Ecological Modelling, 199, pp. 115-124, 2006

Finland (1), USA (3), Australia (3), Israel (2)

AU Perspective

New Secret Weapon Against Severe Uncertainty

$$\hat{\alpha}(q) := \max\{\alpha \geq 0 : r \leq R(q,u), \forall u \in U(\alpha, \tilde{u})\}, q \in \mathcal{Q}$$

Known as

Info-Gap Robustness Model

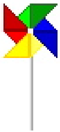
Ben-Haim (1996, 2001, 2006)

Very popular in a number of research organizations in Australia

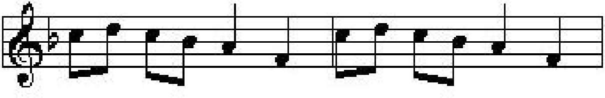
This seminar

Objective of this seminar

- Overview of classical decision theory (1945-50)
- Overview of Robust Decision-Making
- Overview of Voodoo Decision-Making
- Discuss the role and place of Info-Gap Decision Theory in robust decision-making
- Report on my Maximin Campaign
- Raise/Answer questions



Classical Decision Theory



Eg.

620-262: Decision Making

A Simple Problem

Good morning Sir/Madam:

I left on your doorstep four envelopes. Each contains a sum of money. You are welcome to open any one of these envelopes and keep the money you find there.

Please note that as soon as you open an envelope, the other three will automatically self-destruct, so think carefully about which of these envelopes you should open.

To help you decide what you should do, I printed on each envelope the possible values of the amount of money (in Australian dollars) you may find in it. The amount that is actually there is equal to one of these figures.

Unfortunately the entire project is under severe uncertainty so I cannot tell you more than this.

Good luck!

Joe.

So What Do You do?

Example

Envelope	*Possible Amount (Australian dollars)*
$E1$	$20, 10, 300, 786$
$E2$	$2, 40000, 102349, 5000000, 99999999, 56435432$
$E3$	$201, 202$
$E4$	200

Vote!

Modeling and Solution

- What is a decision problem ?
- How do we model a decision problem?
- How do we solve a decision problem?

Decision Tables

Think about your problem as a table, where

- rows represents decisions
- columns represent the relevant possible states of nature
- entries represent the associated payoffs/rewards/costs

Example

Env	*Possible Amount ($AU)*				
$E1$	20	10	300	786	
$E2$	2	4000000	102349	500000000	56435432
$E3$	201	202			
$E4$	200				

Classification of Uncertainty

Classical decision theory distinguishes between three levels of uncertainty regarding the state of nature, namely

- Certainty
- Risk
- Strict Uncertainty

Terminology:

Strict Uncertainty ≡ Severe Uncertainty
≡ Ignorance
≡ True Uncertainty
≡ Knightian Uncertainty
≡ Deep
≡ Extreme
≡ Hard
≡ Fundamental

Severe Uncertainty

Classical decision theory offers two basic principles for dealing with severe uncertainty, namely

- Laplace's Principle (1825)
- Wald's Principle (1945)

Conceptually:

Certainty $\overset{\text{Wald}}{\Longleftarrow}$ **Severe Uncertainty** $\overset{\text{Laplace}}{\Longrightarrow}$ Risk

$\Downarrow$

Voodoo Decision Theory
Info-Gap Decision Theory
???

Laplace's Principle of Insufficient Reason (1825)

Assume that all the states are equally likely, thus use a uniform distribution function (μ) on the state space and regard the problem as decision-making under risk.

Laplace's Decision Rule

$$\max_{d\in\mathbb{D}} \int_{s\in S(d)} r(s,d)\mu(s)\,ds \qquad \text{Continuous case}$$

$$\max_{d\in\mathbb{D}} \frac{1}{|S(d)|}\sum_{s\in S(d)} r(s,d) \qquad \text{Discrete case}$$

Wald's Maximin Principle (1945)

Inspired by Von Neumann's [1928] Maximin model for 0-sum, 2-person games: Mother Nature is and adversary and is playing against you, hence apply the worst-case scenario. This transforms the problem into a decision-making under certainty.

Nice Plain Language Formulation

The maximin rule tells us to rank alternatives by their worst possible outcomes: we are to adopt the alternative the worst outcome of which is superior to the worst outcome of the others.

Rawls, J., *Theory of Justice*, 1971, p. 152

Wald's Maximin Principle (1945)

Historical perspective

The gods to-day stand friendly, that we may,
Lovers of peace, lead on our days to age!
But, since the affairs of men rests still incertain,
Let's reason with the worst that may befall.

William Shakespeare (1564-1616)
Julius Caesar, Act 5, Scene 1

Classic Format

You! Mama

$$\max_{d \in \mathbb{D}} \min_{s \in S(d)} f(d,s)$$

About Maximin/Minimax formulations

Classical Format

$$\underset{d\in\mathbb{D}}{\overset{\text{You!}}{\max}} \ \underset{s\in S(d)}{\overset{\text{Mama}}{\min}} \ f(d,s)$$

Mathematical Programming Format

$$\underset{\substack{d\in\mathbb{D}\\ v\in\mathbb{R}}}{\overset{\text{You!}}{\max}} \left\{ v : f(d,s) \geq v \ , \ \overset{\downarrow \ \text{Mama}}{\forall s \in S(d)} \right\}$$

Note: if $S(d)$ is "continuous", then this is a semi-infinite program.

Laplace vs Wald

Example

Env	*Possible Amount ($AU)*				
$E1$	20	10	300	786	
$E2$	2	4000	102349	50000	56435
$E3$	201	202			
$E4$	200				

Example

Env	*Possible Amount ($AU)*					*Laplace*	*Wald*
$E1$	20	10	300	786		279	10
$E2$	2	4000	10234	50000	56435	24134.2	2
$E3$	201	202				201.5	201
$E4$	200					200	200

Laplace vs Wald

Example

Env	*Possible Amount ($AU)*					*Laplace*	*Wald*
$E1$	20	10	300	786		279	10
$E2$	2	4000	10234	50000	56435	24134.2	2
$E3$	201	202				201.5	201
$E4$	200					200	200

Robust Decision-Making

WIKIPEDIA

Robustness is the quality of being able to withstand stresses, pressures, or changes in procedure or circumstance. A system, organism or design may be said to be "robust" if it is capable of coping well with variations (sometimes unpredictable variations) in its operating environment with minimal damage, alteration or loss of functionality.

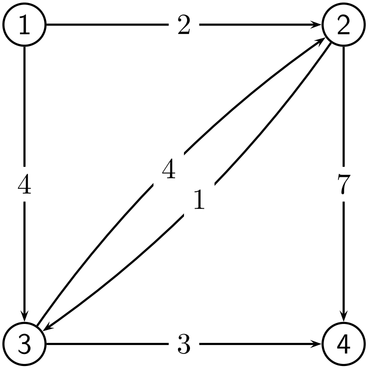
- Applies to both (known) variability and uncertainty
- Origin: probably late 1920's (game theory).
- In OR and Optimization: late 1960s early 1970s.
- Major difficulty: solution procedures.
- A very "hot" area of research these days . . .
- See bibliography

Robust Optimization

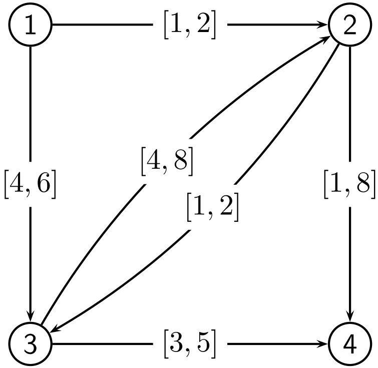
Simple Example

Shortest path problem with variable arc lengths

"Conventional version"



"Robust version"



Robust Decision-Making

Role of Maximin/Minimax in Robustness Analysis

But as we defined robustness to mean insensitivity with regard to small deviations from assumptions, any quantitative measure of robustness must somehow be concerned with the maximum degradation of performance possible for an ϵ-deviation from the assumptions. The optimally robust procedure minimizes this degradation and hence will be a minimax procedure of some kind.

Huber (1981, pp. 16-17)

Experience: Modeling aspects can be subtle!

- Optimizing vs Satisficing
- Complete vs Partial vs Local
- (Mis) Interpretation

Robust Decision-Making

Classification

- Robust Satisficing (eg. Soyster (1973), Ben-Tal and Nemirovski (1999))
  Robustness with respect to constraints of a satisficing problem or an optimization problem.
- Robust Optimizing (eg. classical Maximin/Minimax)
  Robustness with respect to the objective function of an optimization problem.
- Robust Optimizing and Satisficing (eg. Ben-Tal and Nemirovski (2002))
  Robustness with respect to both the objective function and constraints of an optimization problem.

Robust Decision-Making

Classification

- Robust Satisficing

$$\text{Problem } P(u), u \in U\text{:}$$
$$\text{Find an } x \in X \text{ such that } g(x, u) \in C$$

- Robust Optimizing

$$\text{Problem } P(u), u \in U \text{ :}$$
$$z^* := \operatorname*{opt}_{x \in X} f(x, u)$$

- Robust Optimizing and Satisficing

$$\text{Problem } P(u), u \in U \text{ :}$$
$$z^* := \operatorname*{opt}_{x \in X(u)} f(x, u)$$

Robust Decision-Making

Robustness á la Maximin

- Robust Optimizing (Classical Maximin (1945))

$$\max_{d\in\mathbb{D}} \min_{s\in S(d)} f(d,s) \equiv \max_{\substack{d\in\mathbb{D}\\ v\in\mathbb{R}}} \{v : f(d,s) \geq v, \forall s \in S(d)\}$$

- Robust Satisficing (eg. Soyster (1973), Ben-Tal and Nemirovski (1999))

$$\max_{d\in\mathbb{D}} \{\beta(d) : g(d,s) \in C, \forall s \in S(d)\} \equiv \max_{d\in\mathbb{D}} \min_{s\in S(d)} \varphi(d,s)$$

$$\varphi(d,s) := \begin{cases} \beta(d) & , \quad g(d,s) \in C \\ -\infty & , \quad g(d,s) \notin C \end{cases}$$

Robust Decision-Making

Robustness á la Maximin

- Robust Optimizing and Satisficing (eg. Ben-Tal and Nemirovski (2002))

$$\max_{\substack{d\in D\\ v\in\mathbb{R}}} \{v : \gamma(d,s) \geq v, g(d,s) \in C, \forall s \in S(d)\}$$

$$\equiv \max_{d\in\mathbb{D}} \min_{s\in S(d)} \psi(d,s)$$

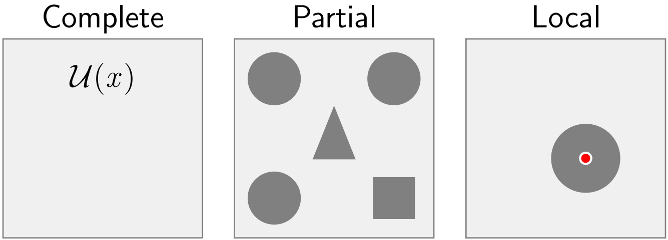
where

$$\psi(d,s) := \begin{cases} \gamma(d,s) & , \quad g(d,s) \in C \\ -\infty & , \quad g(d,s) \notin C \end{cases}$$

Robust Decision-Making

Degree of Robustness

- Complete (conventional)
  $\forall u \in \mathcal{U}(x)$ (very conservative)
- Partial (eg. Starr (1962), Schneller and Sphicas (1983))
  $\forall u \in U(x) \subseteq \mathcal{U}(x)$
- Local (eg. Ben-Haim (2001, 2006, 2008))
  $\forall u \in U(x, \tilde{u}) \subseteq \mathcal{U}(x) \quad (U(x, \tilde{u}) =$ neighborhood of $\tilde{u}$)



Robust Decision-Making

Robustness á la Maximin

Complete robustness

$$z^* := \max_{d \in \mathbb{D}} \min_{s \in S(d)} f(d,s)$$

$$= \max_{\substack{d \in \mathbb{D} \\ v \in \mathbb{R}}} \{v : f(d,s) \geq v, \forall s \in S(d)\}$$

Robust Decision-Making

Robustness á la Maximin

Partial robustness

$$\rho(U) = \text{"size" of set } U$$

$$z^* := \max_{\substack{d\in\mathbb{D}\\ U\subseteq S(d)}} \{\rho(U) : f(d,s) \in C(d,s), \forall s \in U\}$$

$$= \max_{\substack{d\in\mathbb{D}\\ U\subseteq S(d)}} \min_{s\in U} g(d,U,s)$$

where

$$g(d,U,s) := \begin{cases} \rho(U) & , \quad f(d,s) \in C(d,s) \\ 0 & , \quad \text{otherwise} \end{cases}$$

Robust Decision-Making

Robustness á la Maximin

Local robustness

$$U(d,\alpha,\tilde{s}) = \text{ neighborhood of "size" } \alpha \text{ around } \tilde{s}$$

$$\alpha^* := \max_{\substack{d\in\mathbb{D}\\ \alpha\geq 0}} \{\alpha : f(d,s)\in C(d,s), \forall s\in U(d,\alpha,\tilde{s})\}$$

$$= \max_{\substack{d\in\mathbb{D}\\ \alpha\geq 0}} \min_{s\in U(d,\alpha,\tilde{s})} g(d,\alpha,s)$$

$$g(d,\alpha,s) := \begin{cases} \alpha & , \ f(d,s)\in C(d,s) \\ -\infty & , \ \text{otherwise} \end{cases}$$

Remark:
This model is local in nature, hence is unsuitable for severe uncertainty.

Voodoo Decision Theory

Voodoo Decision Theory

Encarta online Encyclopedia

Voodoo n

1. A religion practiced throughout Caribbean countries, especially Haiti, that is a combination of Roman Catholic rituals and animistic beliefs of Dahomean enslaved laborers, involving magic communication with ancestors.
2. Somebody who practices voodoo.
3. A charm, spell, or fetish regarded by those who practice voodoo as having magical powers.
4. A belief, theory, or method that lacks sufficient evidence or proof.

Voodoo Decision Theory

Voodoo Decision Theory

Voodoo Decision Theory

Voodoo Decision Theory

Voodoo Decision Theory

Apparently very popular,

Example

The behavior of Kropotkin's cooperators is something like that of decision makers using Jeffrey expected utility model in the Max and Moritz situation. Are ground squirrels and vampires using voodoo decision theory?

Brian Skyrms
Evolution of the Social Contract
Cambridge University Press, 1996.

Issue:

Evidential dependence, but causal independence.

The legend

An old legend has it that an ancient treasure is hidden in an Asian-Pacific island.

You are in charge of the treasure hunt. How would you plan the operation?

The legend

Main issue: location, location, location!

Terminology

Certainty

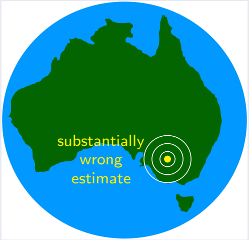
Risk

Severe Uncertainty

Voodooism

The Fundamental Theorem of Voodoo Decision Making

$\approx$



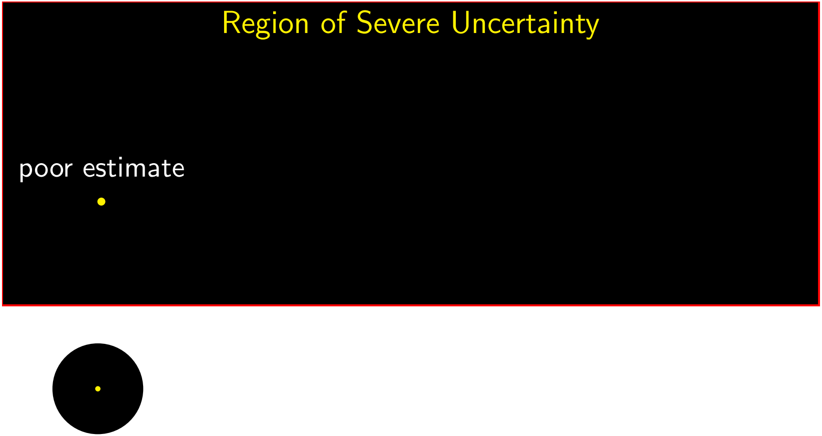
Severe Uncertainty

1·2·3 Recipe

1. Ignore the severe uncertainty.
2. Focus on the substantially wrong estimate you have.
3. Conduct the analysis in the immediate neighborhood of this estimate.

Voodooism

Voodoo Decision-Making



Voodooism

Voodoo Decision-Making

Just in case, . . . , the difficulty is that

Under severe uncertainty

The estimate we have is

- A wild guess.
- A poor indication of the true value.
- Likely to be substantially wrong.

Hence,

Beware!

Results obtained in the neighborhood of the estimate are likely to be substantially wrong in the neighborhood of the true value.

Voodooism

The Curse of Preference Reversal

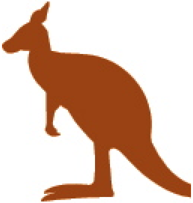
Region of Severe Uncertainty

poor estimate

Plan A is great!!
Plan B is a lemon!!

Plan A is a lemon!!
Plan B is great!!

true value



VS

Voodooism

Conventional Decision Theory

$$\text{GI} \longrightarrow \boxed{\mathfrak{Model}} \longrightarrow \text{GO}$$

$$\text{Wrong} \longrightarrow \boxed{\mathfrak{Model}} \longrightarrow \text{Wrong}$$

The robustness of any decision and the risk incurred in making that decision is only as good as the estimates on which it is based. Making estimation even more challenging, virtually all estimates that affect decisions are uncertain. Uncertainty can not be eliminated, but it can be managed.

Top Ten Challenges for Making Robust Decisions
The Decision Expert Newsletter, Volume 1; Issue 2
http://www.robustdecisions.com/newsletter0102.php

Voodooism

V[illegible] Decisi[illegible]ory

$$\text{GI} \longrightarrow \boxed{[illegible]} \longrightarrow \text{GO}$$

$$\text{Wrong} \longrightarrow \boxed{[illegible]} \longrightarrow \text{Wrong}$$

Voodooism

Voodoo Decision Theory

garbage

$GI \longrightarrow$ **Model** $\longrightarrow GO$

gold

Wrong $\longrightarrow$ **Model** $\longrightarrow$ **Right**

Alchemy

Info-Gap Revisited

Impressive Self-Portrait

Info-gap decision theory is radically different from all current theories of decision under uncertainty. The difference originates in the modelling of uncertainty as an information gap rather than as a probability. The need for info-gap modeling and management of uncertainty arises in dealing with severe lack of information and highly unstructured uncertainty.

Ben-Haim [2006, p. xii]

In this book we concentrate on the fairly new concept of information-gap uncertainty, whose differences from more classical approaches to uncertainty are real and deep.

Ben-Haim [2006, p. 11]

Info-Gap

Obvious Questions

1. Does Info-Gap substantiate these very strong claims?
2. Are these claims valid?

Not So Obvious Answers

1. No, it does not.
2. Certainly not.

It is therefore important to subject Info-Gap to a formal analysis – that actually should have been done seven years ago:

Info-Gap

Formal vs Analysis

Classical Decision Theory

Good news: should take no more than 5-10 minutes!

Info-Gap

Meaning of Severe Uncertainty

- The region of uncertainty is usually relatively large, often unbounded.
- The uncertainty cannot be quantified by a probabilistic model.
- If there is an estimate of the parameter of interest, then the estimate is
  - A wild guess
  - A poor indication of the true value
  - Likely to be substantially wrong

Info-Gap

Practical Meaning of Severe Uncertainty

bio-security homeland-security

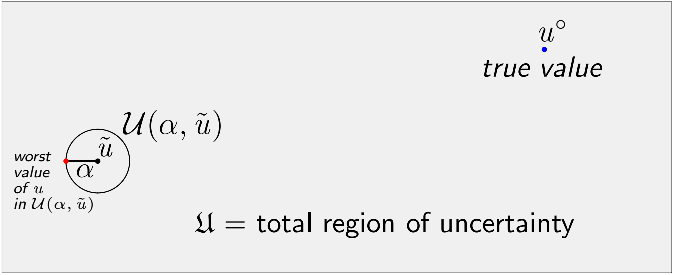
Info-Gap Decision Theory

Complete Generic Robustness Model

$$\hat{\alpha}(r_c) := \max_{q \in \mathbb{Q}} \max \left\{ \alpha \geq 0 : r_c \leq \min_{u \in \mathcal{U}(\alpha, \tilde{u})} R(q,u) \right\}$$

$$\mathcal{U}(\alpha, \tilde{u}) \subseteq \mathcal{U}(\alpha + \varepsilon, \tilde{u}), \forall \varepsilon > 0$$

Region of Severe Uncertainty, $\mathfrak{U}$



Info-Gap Decision Theory

Complete Generic Robustness Model

$$\hat{\alpha}(r_c) := \max_{q \in \mathbb{Q}} \ \max \left\{ \alpha \geq 0 : r_c \leq \min_{u \in \mathcal{U}(\alpha, \tilde{u})} R(q, u) \right\}$$

Fundamental FAQs

1. Is this new? — Definitely not!
2. Is this radically different? — Definitely not!
3. Does it make sense? — Definitely not!

So what is all this hype about Info-Gap ?!

Good question!

Info-Gap Decision Theory

First Impression

Complete Generic Robustness Model

$$\hat{\alpha}(r_c) := \max_{q\in\mathbb{Q}} \ \max\left\{\alpha \geq 0 : r_c \leq \min_{u\in\mathcal{U}(\alpha,\tilde{u})} R(q,u)\right\}$$

Observations

- This model does not deal with severe uncertainty, it simply and unceremoniously ignores it.
- The analysis is invariant with $\mathfrak{U}$: the same solution for all $\mathfrak{U}$ such that $\mathcal{U}(\hat{\alpha}(r_c),\tilde{u}) \subseteq \mathfrak{U}$.
- This model is fundamentally flawed.
- This model advocates voodoo decision-making.

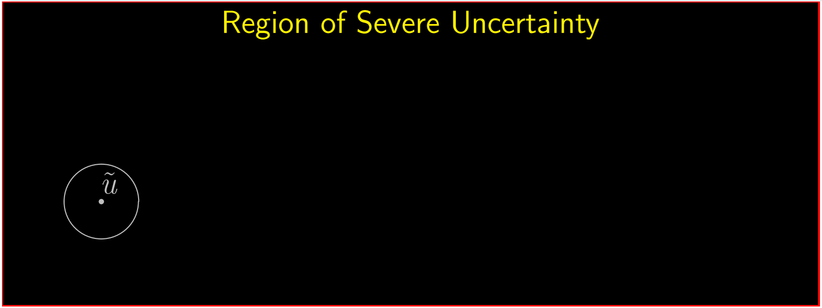
Info-Gap Decision Theory

First Impression

Fool-Proof Recipe

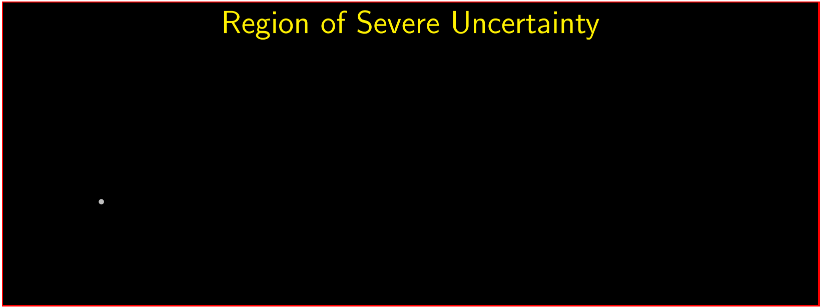
Step 1: *Ignore the severe uncertainty.*

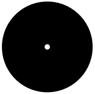
Step 2: *Focus instead on the poor estimate and its immediate neighborhood.*



Info-Gap Decision Theory

First Impression





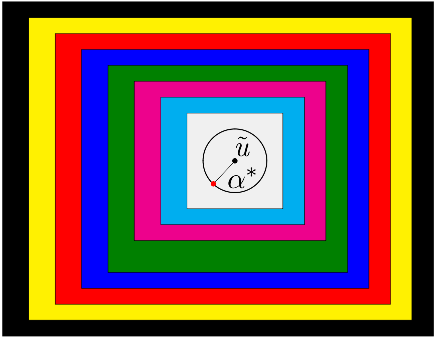
Recall that this is voodoo decision making!

Info-Gap Decision Theory

Complete Generic Robustness Model

$$\alpha^* := \max_{q\in\mathbb{Q}} \ \max\left\{\alpha \geq 0 : r_c \leq \min_{u\in\mathcal{U}(\alpha,\tilde{u})} R(q,u)\right\}$$

Fundamental Flaw



Info-Gap Decision Theory

More formally

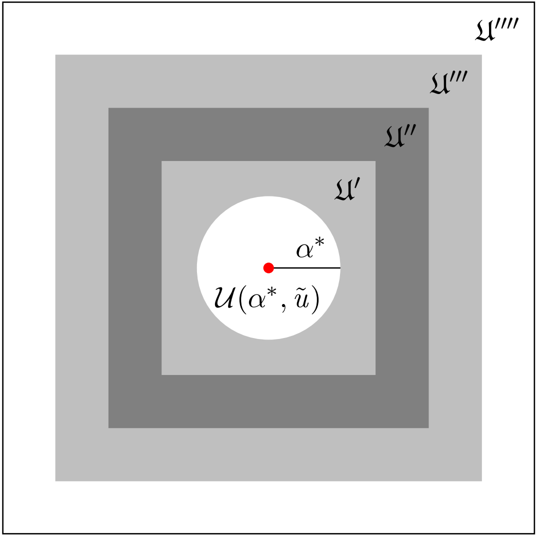
Invariance Theorem (Sniedovich, 2007)

Info-Gap's robustness model is invariant to the size of the total region of uncertainty $\mathfrak{U}$ for all $\mathfrak{U}$ larger than $\mathcal{U}(\alpha^*, \tilde{u})$, where $\alpha^* := \hat{\alpha}(r_c)$.
That is, the model yields the same results for all $\mathfrak{U}$ such that

$$\mathcal{U}(\alpha^* + \varepsilon, \tilde{u}) \subseteq \mathfrak{U} \;,\; \varepsilon > o$$

Info-Gap Decision Theory

Info-Gap's Invariance Property



Info-Gap Decision Theory

Maximin Theorem (Sniedovich 2007, 2008)

Info-Gap's robustness model is a simple instance of Wald's Maximin model. Specifically,

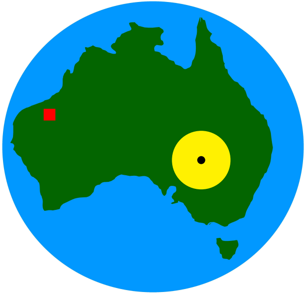
$$\begin{aligned}\alpha(q) &:= \max_{\alpha\geq 0}\left\{\alpha : r_c \leq \min_{u\in\mathcal{U}(\alpha,\tilde{u})} R(q,u)\right\}\ ,\ \ q\in\mathbb{Q}\\ &= \max_{\alpha\geq 0}\min_{u\in\mathcal{U}(\alpha,\tilde{u})}\psi(q,\alpha,u)\end{aligned}$$

where

$$\psi(q,\alpha,u) := \begin{cases}\alpha\ , & r_c\leq R(q,u)\\ 0\ , & r_c > R(q,u)\end{cases}\ ,\alpha\geq 0, q\in\mathbb{Q}, u\in\mathcal{U}(\alpha,\tilde{u})$$

Info-Gap: Typical misconception

Treasure Hunt



Myth:

How wrong can I be, yet be safe?

- Region of uncertainty.
- Estimate of the location.
- Region affecting Info-Gap's analysis.
- True (unknown) location.

Fact:

Info-gap may conduct its robustness analysis in the vicinity of Brisbane (QLD), whereas for all we know the true location of the treasure may be somewhere in the middle of the Simpson desert or perhaps in down town Melbourne (VIC). Perhaps.

Australian Perspective

Conclusions

- Decision-making under severe uncertainty is difficult.
- It is a thriving area of research/practice.
- The Robust Optimization literature is extremely relevant.
- The Decision Theory literature is extremely relevant.
- The Operations Research literature is very relevant.
- Info-Gap's robustness model is neither new nor radically different.
- Info-Gap's uncertainty model is fundamentally flawed and is unsuitable for decision-making under severe uncertainty.
- Info-Gap Decision Theory exhibits a severe information-gap about the state of the art in decision-making under severe uncertainty.

FAQs?

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