Working Paper No. MS-03-08

The Info-Gap/Maximin Saga Continues . . .

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June 5, 2008

Last update: December 18, 2011

How to read this paper

· This is a self-contained discussion on the relationship between Ben-Haim's (2001, 2006) Info-Gap decision theory and Wald's (1945, 1950) Maximin paradigm. It was was prompted by a recent erroneous off-the-cuff commentary on this relationship in Los and Tungsong (2008).

· Consult the relevant section(s) in the Appendix for background information/material on the topics discussed in the main body of the paper.

· Don't worry about the seemingly excessive amount of time/effort that I spent on explaining the obvious errors in Los and Tungsong (2008) regarding the Info-Gap/Maximin connection. I have good reasons for doing this and I . . . greatly enjoy this project!

· To keep some of the sections self contained, a number of issues are discussed twice – at different level of detail.

· You are advised that my criticism of Ben-Haim's (2001, 2006) Info-Gap decision theory is harsh. The degree of harshness is proportional to the seriousness of the flaws in this theory and to the level of promotion it is attracting in Australia and elsewhere.

· My website (www.moshe-online.com) provides additional resources relevant to this project.

· I offer exciting, stimulating, thought provoking, entertaining presentations, seminars, lectures, and workshops on this and related topics, including

The Rise and Rise of Voodoo Decision Theory

· As always, Info-Gap aficionados are welcome!

Abstract

On numerous occasions and in many articles I have formally shown (proved) that Info-Gap's robustness model (Ben-Haim 1999, 2001, 2006) is a Maximin model (Wald 1945, 1950) par excellence and that it is fundamentally flawed in the way it handles severe uncertainty.

Furthermore, I have shown that the Info-Gap/Maximin connection is not strictly mathematical in nature. Indeed, the conceptual framework of Info-Gap's robustness model is the same as that of the instance of the Maximin model representing it, namely it is grounded in a typical "worst-case analysis" approach to uncertainty. The difference is in the terminology and jargon.

I was therefore surprised to learn that in a recent article Los and Tungsong (2008) raise some doubts regarding the validity of my criticism in Sniedovich (2007) of Info-Gap decision theory.

The good news is that Los and Tungsong's (2008) off-the-cuff commentary on my views on the Info-Gap/Maximin connection and the local nature of Info-Gap's robustness model is . . . totally wrong.

The bad news is that what we have here is a completely uncalled for academic style "comedy of errors", one that I take seriously, hence this paper. I do hope that, as in the case of the original (Shakespeare 1594) play, there will be a happy-end here as well!

In any case, Los and Tungsong (2008) use the term "information gap" very, very loosely and with no apparent reason they assume that my criticism of Ben-Haim's (2001, 2006) Info-Gap decision theory also applies to information gap models/theories that are not compliant with the structure of the model specified in . . . Ben-Haim (2001, 2006).

In other words, I criticize a very specific information gap model, namely Ben-Haim's (2001, 2006) model, whereas Los and Tungsong (2008) misconstrue this criticism by assuming that it applies to all information gap models in this Universe.

I have no idea what prompted Los and Tungsong (2008) to commit this blatant, uncalled for error, but they did.

The objective of this paper is to fix the blunder on the part of Los and Tungsong (2008), and to explain, once more, the Info-Gap/Maximin connection.

It turns out, however, that this "comedy of errors" does have some positive contribution to the state of the art in decision theory: it confirms the validity of my harsh criticism of Ben-Haim's (2001, 2006) Info-Gap decision theory.

This entire episode, therefore, reminds me of Samson's very famous riddle:

> *Out of the eater came something to eat*
> *and out of the strong came something sweet.*
> *Judges 14:14*

Keywords: Maximin, Info-Gap decision theory, Knightian uncertainty, Robustness, Local, information-gap, voodoo decision theory.

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Read Me First

Dear Reader:

It is extremely important that you note and appreciate the fact that the terms

- Info-Gap
- Info-Gap decision theory
- Info-Gap's robustness model

that I use in this discussion – and in all my articles and presentations on this topic over the past two years – refer explicitly and exclusively to the decision theory described in Ben-Haim (2001, 2006).

So for example, if I say that "Info-Gap is a voodoo decision theory", this is short for "Ben-Haim's (2001, 2006) Info-Gap decision theory is a voodoo decision theory".

I mention this fact even though I am not aware of the existence of any other decision theory called "Info-Gap" and even though I always make it crystal clear – via standard referencing and citation conventions – that the theory that I analyze is Ben-Haim's (2001, 2006) Info-Gap decision theory.

In particular, in Sniedovich (2007, Introduction, p. 111) I note:

> The aim of this paper is to illustrate that the mathematical modeling of decision-making under severe uncertainty requires considerable subtlety.
>
> For this purpose we use Info-Gap (Ben-Haim, 2006) as a case study.

I issue this public clarification here to guard against a possible confusion that may occur between the above terms and terms such as

- Information gap
- Information gap uncertainty
- Information gap model

that are used by other authors, eg. Los and Tungsong (2008), in reference to theories that are different from Ben-Haim's (2001, 2006) Info-Gap decision theory, and the existence/nonexistence of which I am not aware.

Enjoy your reading!

Moshe
Melbourne, Australia
July 14, 2008

1 Introduction

Over the past four years I have expressed my harsh criticism of Ben-Haim's (2001, 2006) Info-Gap decision theory in public and in private. This criticism is well document now: see my website `info-gap.moshe-online.com` and `WIKIPEDIA` for details.[1]

The focus of my criticism is Ben-Haim's (2001, 2006) generic robustness model, namely[2]

$$\hat{\alpha}(q, r_c) := \max\ \{\alpha \geq 0 : r_c \leq R(q,u), \forall u \in \mathcal{U}(\alpha, \tilde{u})\}\ ,\ q \in \mathcal{Q} \tag{1}$$

In other words, my criticism is not "abstract" in nature: it refers to a very specific, concrete mathematical model.

Now, there are many flaws in Ben-Haim's (2001, 2006) Info-Gap decision theory and in Sniedovich (2007) my criticism is primarily concerned with the following two properties of the robustness model shown in (1):

· This model is local in nature and therefore it does not tackle the assumed severity of the uncertainty under consideration. It simply ignores it.

· This model is a simple instance of Wald's (1945, 1950) very famous Maximin model, namely

$$z^* := \max_{a \in A} \min_{s \in S(a)} f(a, s) \tag{2}$$

With regard to the Maximin connection, the flaw is not that (1) is a Maximin model. After all, the overwhelming majority of the robustness models in decision-making under severe uncertainty are Maximin models or related models.

But this is precisely the point: the flaw is in Ben-Haim's (2001, 2006) assessment of the role and place of Info-Gap decision theory in decision theory in general and decision-making under severe uncertainty in particular. Specifically, the contention that Ben-Haim's (2001, 2006) robustness model (1) is radically different – methodologically speaking – from Wald's Maximin model (2) is groundless.

In fact, the opposite is true: (1) is a simple instance of (2). To wit: it is easy to show that under the conditions stipulated by Ben-Haim's (2001, 2006) decision theory,

$$\max\ \{\alpha \geq 0 : r_c \leq R(q,u), \forall u \in \mathcal{U}(\alpha, \tilde{u})\} = \max_{\alpha \geq 0} \min_{u \in \mathcal{U}(\alpha, \tilde{u})} \alpha \cdot (r_c \preceq R(q,u)) \tag{3}$$

where the binary operation $\preceq$ is defined as follows

$$a \preceq b := \begin{cases} 1 & , \quad a \leq b \\ 0 & , \quad a > b \end{cases} \qquad a, b \in \mathbb{R} \tag{4}$$

where $\mathbb{R}$ denotes the real line.

In any case, through the use of standard reference and citation conventions I always make it crystal clear that the model that I criticize is the one described by Ben-Haim (2001, 2006). The presentation in Sniedovich (2007) is a case in point.

[1] `info-gap.moshe-online.com` and `www.wikipedia.com/wiki/info-gap_decision_theory`

[2] See details in the appendix.

I was therefore surprised to learn very recently that for some unexplained reason Los and Tungsong (2008) completely misconstrue the scope of the criticism in Sniedovich (2007) by assuming that my criticism is directed at any model/theory that makes use of the buzz word "information gap".

I have no idea what has caused this major, uncalled for mishap.

To see more clearly the "comedy of errors" aspect of the situation, note the (communication) gap between the following two statements made by two fictitious characters:

· Sam: *"The green apple on the desk is rotten!"*

· Carl: *"Sam claims that any apple in Australia is rotten!"*

By analogy:

· I criticize explicitly and exclusively Ben-Haim's (2001, 2006) generic Info-Gap robustness model (1).

· Los and Tungsong (2008) claim that I criticize any model/theory that uses the term "information gap".

Obviously, I can dismiss Los and Tungsong's (2008) commentary simply as an unfortunate misinterpretation of the scope of my discussion in Sniedovich (2007) and move to other projects listed on my ToDo sticker.

Yes I can. But I shall not.

The objective of this discussion is not to try to figure out what caused Los and Tungsong (2008) to so grossly err in their analysis of the target of my criticism. Rather, the main goal here is to explain the flaws in Los and Tungsong's (2008) off-the-cuff commentary on the Info-Gap/Maximin connection.

The paper consists of two parts:

· In the main body I examine the errors in Los and Tungsong's (2008) commentary on the Info-Gap/Maximin connection and discuss the three buzz words associated with this saga: Knightian uncertainty, information gap, and Maximin

· In the appendix I explain, once more, my criticism of Ben-Haim's (2001, 2006) Info-Gap decision theory.

If you are not familiar with Ben-Haim's (2001, 2006) Info-Gap decision theory, it is the right time to consult the appendix.

2 What's wrong in Los and Tungsong (2008)?

I regard Los and Tungsong's (2008) commentary on the Info-Gap/Maximin connection as a good example of the danger in engaging in off-the-cuff commentary, in posting in the public domain discussions on half-baked ideas, and in the ever increasing temptation (peer pressure?) to use trendy buzz words. In the case of Los and Tungsong (2008) the buzz words are "information gap" and "Knightian uncertainty".

I imagine that the excessive use of these terms in Los and Tungsong (2008) is a sort of compensation for the complete absence of these terms in Los (1999, 2003, 2006).

And now to the commentary itself.

1. It looks like the following quote – including its footnote – prepares the grounds for a claim to the term "information gap".

   > My[sic] Galton's Error critique of the conventional bivariate Capital Asset Pricing Model (CAPM) based investment decision-making showed that, when there is serious information gap,[1] most investment decision-makers prefer to ignore this lack of information, providing evidence of their ambiguity aversion (a term ascribed to Fox and Tversky, 1995).
   >
   > [1]In my[sic] Los (1999) article, I[sic] called it the "ignorance gap," but that is, strictly speaking, incorrect since we are dealing with a gap in our information or knowledge, not a gap in our ignorance. However, the measurement of our information gap is a, skeptical but plausible, measurement of our ignorance
   >
   > Los and Tungsong (2008, p. 1)

   The funny thing is that tried as I did, I could not find the term "ignorance gap" in Los's (1999) paper. I did find there, though, the term "uncertainty gap". This term appears twice, in both cases on page 1811, in connection with the value of a parameter θ_2 that represents modeling uncertainty (Los (1999, p. 1809). More specifically, the first appearance is as follows:

   > "(iv) $\theta_2 = 0$, there exists no uncertainty gap between the orthogonal frames of data reference;"

   and nine lines below the second appearance reads as follows:

   > "(iv) $0 < \theta_2 < \pi/2$, there exists an uncertainty gap between the orthogonal frames of data reference;"

   And that's it folks!

   In comparison, in Los and Tungsong (2008, p. 7) these very ideas are expressed as follows:

   > "(iv) $\theta_2 = 0$, there exists no information gap"

   and 6 lines later

   > "(iv) $0 < \theta_2 < \pi/2$, there exists an infomation gap"

   respectively.

2. Los and Tungsong (2008) commentary on the Info-Gap/Maximin connection refers to my 2007 paper (Sniedovich 2007). Unfortunately, Los and Tungsong (2008) did not read my paper carefully. For had they done so they would not have written the following regarding the buzz word *information-gap* :

> Only recently I [sic] learned that mathematicians Ben-Haim and Sniedovich are currently probing similar non-probabilistic decision-making theory issues more generally (Ben-Haim, 2006; Sniedovich, 2007).
>
> Los and Tungsong (2008, p. 2)

In Sniedovich (2007) I am not "probing" non-probabilistic decision-making theory issues "more generally". What I do there is precisely what I do in this paper: I discuss the art and science of modeling decision making under severe uncertainty and I criticize Ben-Haim's (2001, 2006) Info-Gap decision theory.

3. Furthermore, on the same page we find the following odd claim:

> Sniedovich (2007) is critical of any information-gap models, which, he asserts, generically resemble Wald's Maximin models (Wald, 1945, 1950) and, therefore, can lead to only locally optimal and, therefore, rationally limited decisions.
>
> Los and Tungsong (2008, p. 2)

Reading this I just wonder if Los and Tungsong (2008) actually read my paper (Sniedovich 2007). So here are some facts:

(a) In Sniedovich (2007) I discuss one, and only one, "info-gap model", namely Ben-Haim's (2001, 2006) "info-gap model".

(b) So the criticism in Sniedovich (2007) refers to one, and only one, info-gap model – not to "any information-gap models". I am not familiar with any information-gap decision theories other than Ben-Haim's (2001, 2006) Info-Gap decision theory.

(c) Ben-Haim's (2001, 2006) decision theory is a VOODOO DECISION THEORY because it conducts its robustness analysis only in the immediate neighborhood of a poor estimate that is likely to be substantially wrong. This has nothing to do with the fact that the analysis is of the Maximin type.

(d) The local nature of Ben-Haim's (2001, 2006) Info-Gap robustness model has nothing to do with it being a Maximin model (Wald 1945, 1950) in disguise. There are Maximin robustness models that are not local in nature. In fact, most of the Maximin models in the literature are not local in nature.

4. And on the very same page we also find this pearl:

> However, this paper demonstrates that this is a very doubtful, if not outrightly wrong, assertion, since information gap decision models do not use a probabilistic measure function: they focus on the incompleteness or lack of information.
>
> Los and Tungsong (2008, p. 2)

Frankly, I do not know what Los and Tungsong (2008) are up to here.

(a) The criticism in Sniedovich (2007) refers explicitly and exclusively to Ben-Haim's (2001, 2006) theory and in this framework the criticism is perfectly valid.

(b) I do not know what "information gap decision models" Los and Tungsong (2008) have in mind: although they use the term "information gap" extensively, they do not mention any specific formal information-gap model in their paper.

(c) In any case, in their paper Los and Tungsong (2008) do not demonstrate that my claims are doubtful. There is no formal treatment of the Info-Gap/Maximin issue in the paper. And in any case, such a demonstration is impossible: there is a formal proof in Sniedovich (2007) that my claim is valid.

5. The saga overflows to the next the page:

> But Wald's Maximin model does not take account of incomplete information about all possible "states of Nature," and of ambiguity aversion and its consequent thirst for scientific R&D, which can expand the known set of "states of Nature" and reduce Sniedovich's "region of severe uncertainty," while Ben-Haim's information gap model does.
>
> Los and Tungsong (2008, pp. 2-3)

As shown in Sniedovich (2007) and in this paper, the Maximin Theorem is very clear about the relationship between Ben-Haim's (2001, 2006) Info-Gap regions and uncertainty and the state spaces of the Maximin model representing Ben-Haim's (2001, 2006) Info-Gap robustness model: they are identical!

So what is this big idea that Ben-Haim's (2001, 2006) uncertainty model can deal with incomplete information while Wald's Maximin model cannot?!?!?!

Here are the two models side by side:

Info-Gap Robustness model		Wald's Maximin Model	
$\max\{\alpha : r_c \leq R(q,u), \forall u \in \mathcal{U}(\alpha,\tilde{u})\}$	$=$	$\max\limits_{\alpha \geq 0} \min\limits_{u \in \mathcal{U}(\alpha,\tilde{u})} \alpha \cdot (r_c \preceq R(q,u))$	(5)

In the framework of the Maximin model the state space (domain of the min operation) associated with alternative α is $\mathcal{U}(\alpha,\tilde{u})$ which is precisely Info-Gap's region of uncertainty (Ben-Haim 2001, 2006) associated with this alternative.

In short, to re-iterate: Ben-Haim's (2001, 2006) Info-Gap robustness model is a simple instance of Wald's (1945, 1950) Maximin model. Hence, whatever Ben-Haim's (2001, 2006) Info-Gap robustness model can do, so can Wald's (1945, 1950) Maximin model – and much more.

In particular, Los and Tungsong (2008) are wrong in claiming that the state space (= uncertainty region) of Ben-Haim's (2001, 2006) robustness model is not fixed in advance. It is fixed in advance.

6. In fact, it is very odd that Los and Tungsong (2008) raise this issue in the first place. After all, they themselves note that:

> An information-gap model does quantify the possible range of uncertainty, but without any measure function.
>
> Los and Tungsong (2008, p. 2)

But "Assuming that all states of Nature are known" is the same thing as "quantifying the possible range of uncertainty".

So what exactly is the issue here?

Wald's (1945, 1950) Maximin model does not require anything that is not required by Ben-Haim's (2001, 2006) Info-Gap robustness model.

7. In a footnote on the same page we find the following claim:

> [3]Sniedovich error of assertion is similar to that of the adherents to the Intelligent Design of the Universe, who presume to know all "states of Nature." Human knowledge is inherently limited, expandable and replaceable, i.e., *incomplete.*
>
> Los and Tungsong (2008, p. 3)

Well, well well: how about this!!!

I can assure the reader that Sniedovich is definitely not an adherent to the *Intelligent Design of the Universe*. What Sniedovich (2007) proves is that

Info-Gap Robustness model		Wald's Maximin Model
$\max\{\alpha : r_c \leq R(q,u), \forall u \in \mathcal{U}(\alpha,\tilde{u})\}$	$=$	$\max_{\alpha\geq 0} \min_{u\in\mathcal{U}(\alpha,\tilde{u})} \alpha \cdot (r_c \preceq R(q,u))$

If Los and Tungsong (2008) do not like the idea that an "information gap" model (whatever it is) should fix the uncertainty space (= state space) a priori, then they should discuss this matter with Ben-Haim (2001, 2006) because this is exactly what Ben-Haim's (2001, 2006) Info-Gap robustness model does. To wit: The regions of uncertainty $\mathcal{U}(\alpha,\tilde{u}), \alpha \geq 0$ associated with Ben-Haim's (2001, 2006) robustness model

$$\hat{\alpha}(q,r_c) := \max\{\alpha : r_c \leq R(q,u), \forall u \in \mathcal{U}(\alpha,\tilde{u})\} \tag{6}$$

are clearly assumed to be known in advance.

The incomplete information is not with regard to these sets but with regard to which element of these sets in the true value of u.

8. Los and Tungsong (2008, p. 20) indicate that their "dynamic" CML-based model, unlike Ben-Haim's (2001, 2006) model, is not based on a point estimate:

> Therefore, the dynamic CML-based investment decision model of Fig. 3 does not rely on a point estimate and, therefore, provides an information-gap model that is not generically equivalent to Wald's Maximin investment decision model. This implies that it escapes the harsh critique of Sniedovich (2007, p. 125) that "the flaw in the Info-Gap uncertainty model" . . . "lies in the use of a single point estimate and its neighborhood as an approximation of an entire region of uncertainty." This expanded CML-based information-gap investment decision model allows for the exploration of thousands of investment opportunities, which dynamically "bubble up" in the average-return-uncertainty space of Markowitz.

There is a serious confusion here between a number of unrelated issues:

- The "Maximin" and "Point Estimate" issues are unrelated. A maximin model does not have to be based on a point estimate.
- The dynamic nature of the model, on its own, does not guarantee that the model is not local in nature. Hence, the model could still end up being local in nature, hence not suitable for conditions of severe uncertainty.
- The severe uncertainty in the "dynamic" aspects of the model increases the severity of the overall uncertainty associated with the model.

- Thus, without knowing the exact details of the formulation of the "bubble-up" process and the uncertainty involved, it is premature to speculate on the performance of the dynamic model.

9. In view of all of this, it seems that Los and Tungsong (2008) have serious misconceptions about the relationship between their notion of how an "information gap" model should be like, and how Ben-Haim's (2001, 2006) concrete "info-gap model" actually looks like.
10. And if they do not like the fact that Ben-Haim's (2001, 2006) Info-Gap decision theory is an adherent to the Intelligent Design of the Universe concept, that it presumes to know all "states of Nature", that it does not prescribe to the idea that human knowledge is inherently limited, expandable and replaceable, i.e., *incomplete*, then ... they should address their criticism to Ben-Haim (2001, 2006), not to Sniedovich (2007).
11. In view of this, Los and Tungsong (2008) should do well to read carefully Ben-Haim (2001, 2006) formulation of Info-Gap decision theory. They will discover that Ben-Haim's (2001, 2006) Info-Gap generic robustness model is as follows:

$$\hat{\alpha}(q, r_c) := \max\{\alpha : r_c \leq R(q,u), \forall u \in \mathcal{U}(\alpha, \tilde{u})\} \ , \ q \in \mathcal{Q} \tag{7}$$

    Furthermore, they will discover that the theorems in Sniedovich (2007) are correct, and therefore that my criticism is not only on target, but long overdue.
12. Los and Tungsong (2008) should be also well advised not to confuse their care-free use of the term "information gap" with the terms "Info-Gap decision theory" used by Ben-Haim (2001, 2006), Sniedovich (2007), and others.

Summary

My assessment is that Los and Tungsong (2008) are new victims of the various misconceptions circulating in the literature regarding the role and place of Ben-Haim's (2001, 2006) Info-Gap decision theory in decision theory and the kind of robustness that it espouses.

Specifically, Los and Tungsong (2008) are wrong in concluding that by some magic secret powers Ben-Haim's (2001, 2006) Info-Gap decision theory is capable of taking care of incomplete information by expanding/contracting the known set of "states of Nature". I ascribe this misconception, to a large extent, to the excessive use of the term "Knightian uncertainty" in the Info-Gap literature.

Talking about Knightian uncertainty, it is interesting to note that the center piece of Ben-Haim's (2001, 2006) Info-Gap decision theory, namely its robustness model, is identical to the robust model presented in Ben-Haim's (1996) book entitled *Robust Reliability in the Mechanical Science* , where there is no mention of the term "Knightian uncertainty" and where it is not assumed that the uncertainty is severe.

Also interesting is the fact that although the term "information gap" does appear a number of times in Ben-Haim's (1996) book, it does not appear in the title of the book.

From the "information gap" perspective, it is not clear at all what is accomplished in Los and Tungsong (2008). What exactly is accomplished by replacing terms such as "model uncertainty" and "uncertainty gap" in Los's (1999) models with more exotic terms such as "Knightian uncertainty" and "information gap"?

3 Anatomy of three buzz words

In this section I take a quick look at three buzz words associated with the Info-Gap/Maximin saga, namely

· Knightian uncertainty

· Information-gap

· Maximin

The first two are very popular in the Info-Gap literature. The third is an anathema there and I take the liberty of adding that I am very pleased that this is so. Furthermore, I am also very pleased that, to a large extent, this is due to my Maximin, Info-Gap and Voodoo decision-making campaigns (`www.moshe-online.com`).

3.1 Knightian uncertainty

This type of uncertainty is named after the economist Frank Hyneman Knight (1885-1972), who was one of the founders of the so-called "Chicago school of economics" and who is credited with the distinction between "risk" and "uncertainty".

> To preserve the distinction which has been drawn in the last chapter between the measurable uncertainty and an unmeasurable one we may use the term "risk" to designate the former and the term "uncertainty" for the latter. The word "risk" is ordinarily used in a loose way to refer to any sort of uncertainty viewed from the standpoint of the unfavorable contingency, and the term "uncertainty" similarly with reference to the favorable outcome; we speak of the "risk" of a loss, the "uncertainty" of a gain. But if our reasoning so far is at all correct, there is a fatal ambiguity in these terms, which must be gotten rid of, and the use of the term "risk" in connection with the measurable uncertainties or probabilities of insurance gives some justification for specializing the terms as just indicated. We can also employ the terms "objective" and "subjective" probability to designate the risk and uncertainty respectively, as these expressions are already in general use with a signification akin to that proposed.
>
> The practical difference between the two categories, risk and uncertainty, is that in the former the distribution of the outcome in a group of instances is known (either through calculation a priori or from statistics of past experience), while in the case of uncertainty this is not true, the reason being in general that it is impossible to form a group of instances, because the situation dealt with is in a high degree unique. The best example of uncertainty is in connection with the exercise of judgment or the formation of those opinions as to the future course of events, which opinions (and not scientific knowledge) actually guide most of our conduct.
>
> Knight (1921, III.VIII.1-2)

Personally, I prefer the following quote from a paper by the famous British economist John Maynard Keynes (1883 - 1946), whose ideas, known as "Keynesian economics", had a major impact on modern economic and political theory.

> By "uncertain" knowledge, let me explain, I do not mean merely to distinguish what is known for certain from what is only probable. The game of roulette is not subject, in this sense, to uncertainty; nor is the prospect of a Victory bond being drawn. Or, again, the expectation of life is only slightly uncertain. Even the weather is only moderately uncertain. The sense in which I am using the term is that in which the prospect of a European war is uncertain, or the price of copper and the rate of interest twenty years hence, or the obsolescence of a new invention, or the position of private wealth owners in the social system in 1970. About these matters there is no scientific basis on which to form any calculable probability whatever. We simply do not know. Nevertheless, the necessity for action and for decision compels us as practical men to do our best to overlook this awkward fact and to behave exactly as we should if we had behind us a good Benthamite calculation of a series of prospective advantages and disadvantages, each multiplied by its appropriate probability, waiting to be summed.
>
> Keynes (1937, pp. 213-5)

The term "Knightian uncertainty" is very popular in the Info-Gap literature. For instance, it appears about 43 times in Los and Tungsong (2008).

But how exactly is the severity of the "Knightian uncertainty" actually manifested in Ben-Haim's (2001, 2006) Info-Gap robustness model?

Let's see.

This is Ben-Haim's (2001, 2006) Info-Gap robustness model:

$$\hat{\alpha}(q, r_c) := \max\{\alpha \geq 0 : r_c \leq R(q, u), \forall u \in \mathcal{U}(\alpha, \tilde{u})\} \ , \ q \in \mathcal{Q} \tag{8}$$

The important thing to note is that

- $\tilde{u}$ denotes the estimate of the parameter of interest whose true value is subject to **severe** uncertainty.

  Under these conditions we have to assume, as Info-Gap decision theory does, that $\tilde{u}$ is

  - A **wild guess**
  - A **poor indication** of the true value of the parameter of interest
  - Likely to be **substantially wrong**.

- According to Ben-Haim (2006, p. 210), the total region of uncertainty is typically **unbounded**, so typically the value of $\hat{\alpha}(q, r_c)$ is minute relative to the size of the total region of uncertainty.

As I have shown on many occasions (see Appendix), this means that in accordance with the GIGO Axiom[3], the results generated by this model are also wild guesses that are likely to be substantially wrong.

It is precisely because of this "local" characteristic of Ben-Haim's (2001, 2006) Info-Gap robustness model that I regard Ben-Haim's (2001, 2006) Info-Gap decision theory as a classical *Voodoo Decision Theory* par excellence.

[3]Garbage In - Garbage Out.

3.2 Information gap

I do not know who coined term "information gap" and when this was done. Many attribute it to J. Galbraith (1973) book *Designing Complex Organizations* (1973). In this book Galbraith discusses, among other things, the importance of the the *disparity* between the information available to an organization and the information needed by the organization to perform its business operations.

However, for the record, try as I did, I could not find the magic term "information gap" in this book.

A quick `Google` and `Amazon` (books) searches immediately reveal how buzzy the term "information gap" is these days. It appeared in the title of a publication at least as early as 1979 and there are numerous books whose titles include this term. In particular, "bridging the information gap" is a very popular title/subtitle of books and articles in many fields.

Very broadly speaking, today "information gap" means "the disparity between two or more information contents".

This, of course, does not say much about the gap itself and what it represents, and this is why the term is such a useful/useless buzz word – depending on the context. And this is precisely why it should be used with care in the context of a discussion of a specific theory that utilizes this term.

Likewise, derived terms such as "information gap model" and "information gap theory" should be use with care, as they can mean different things to different people and/or in different contexts.

For example, consider the meaning of "information gap model" in the following quote:

> The 'information gap' model may need to be redefined in more culturalist terms than those of its current formulation, as cultural barriers also have very material effects.
>
> David Morley
> *Television, audiences and cultural studies*
> Routledge, 1992 (p. 218)

What exactly is the "information gap model" in this context?

And how about the meaning of the term "information gap theory" in the following three related quotes:

> Right-Handed Cats and the Gap Theory of Curiosity
>
> Are cats right- or left-pawed? Do they favor a paw the way we favor a hand?
>
> If you're like me, this question made you curious. So let's leave the world of cats for a second and consider the meta-question: What kinds of things make people curious?
>
> Psychologists have wrestled with this mystery for many years. In 1994, George Loewenstein, a behavioral economist at Carnegie Mellon, came up with a theory of curiosity. He called it the "information-gap theory."
>
> He said that curiosity is simple: It comes when we feel a gap between what we know and what we want to know. And he goes further: He said that the gap actually causes us a kind of pain ? like an itch that

> we need to scratch. And that's where the "fire" of curiosity comes from ? we are driven to fill the gap, to scratch the itch.
>
> Chip and Dan Heath
> February 20th, 2007
> Powell's Books
> http://www.powells.com/blog/?p=1854

> Consistent with this view, the information-gap theory views curiosity as arising when attention becomes focused on a gap in one's knowledge. Such information gaps produce the feeling of deprivation labeled curiosity. The curious individual is motivated to obtain the missing information to reduce or eliminate the feeling of deprivation.
>
> Loewenstein (1994, p. 87)

> Lack of curiosity about others as a result of the failure to recognize information gaps may be a contributing factor to the well-documented resistance of stereotypes to change. At the same time, however, the information-gap theory suggests a possible solution to the problem. If people are made aware of their stereotypes and of the predictions they make on the basis of them, they may become curious to know whether their predictions are correct.
>
> Loewenstein (1994, p. 94)

What exactly is Loewenstein's (1994) information-gap theory?

In short, to-reiterate, terms such as "information gap model" and "information gap theory" should be use with care, as they can mean different things in different contexts.

This is why I am very careful to make it crystal clear that my Info-Gap Campaign deals explicitly and exclusively with Ben-Haim's (2001, 2006) Info-Gap decision theory.

More specifically, this is why I am very careful to make it crystal clear that my Maximin, Info-Gap, and Voodoo Decision-Making Campaigns (www.moshe-online.com) are about the flaws in Ben-Haim's (2001, 2006) Info-Gap decision theory whose generic robustness model is as follows:

$$\hat{\alpha}(q, r_c) := \max \{\alpha \geq 0 : r_c \leq R(q, u), \forall u \in \mathcal{U}(\alpha, \tilde{u})\} \ , \ q \in \mathcal{Q} \tag{9}$$

For some inexplicable reason, although Los and Tungsong (2008) are fully aware of this fact, they take the liberty of assuming – erroneously – that in Sniedovich (2007) my criticism of Ben-Haim's (2001, 2006) Info-Gap decision theory applies to any information gap models.

Back to the buzzword.

The thing to note, though, is that as buzzy as the term "information gap " is these days in the Info-Gap literature, it possesses no magic powers and is not capable, on its own, to overcome the challenges of decision-making under severe uncertainty.

Replacing the term "uncertainty gap", or the term "discrepancy" with the term "information gap" in a description of a model in an old paper will not change the model itself, nor its contribution to the state of the art.

As far as science is concerned, what is important is the substance that a term represents, not its current level of buzziness. To paraphrase a colleague, " it is perfectly OK these days to include some spin in publications and grant applications, but there should be some substance behind the spin".

In view of the extensive use of this term in Los and Tungsong (2008), it is appropriate to have a look at how it is used and what it represents in the framework of Ben-Haim's (2001, 2006) Info-Gap decision theory.

So recall that in the context of Ben-Haim's (2001, 2006) Info-Gap decision theory, this term refers to the DISPARITY between the ESTIMATE we have of the parameter of interest, and the TRUE (unknown) value of this parameter.

Thus, within the scope of Ben-Haim's (2001, 2006) Info-Gap decision theory – where the uncertainty in the true value of the parameter of interest is assumed to be SEVERE – this "gap" is subject to SEVERE uncertainty.

It should be pointed out, however, that other than using a catchy term to describe a mundane quantity – the disparity between an estimate and the true value of the parameter of interest – Ben-Haim's (2001, 2006) Info-Gap decision theory, more specifically its robustness model, does not grapple at all with the quantification of the uncertainty as such.

In other words, in the context of Ben-Haim's (2001, 2006) Info-Gap decision theory, the term "information gap" itself does not represent any approach (new or old) to the old and difficult problem: quantification and management of severe uncertainty. As indicated above, the term simply means "the DISPARITY between the ESTIMATE and the TRUE VALUE of the parameter of interest".

So given that the term "information gap" itself and what it represents is no silver bullet for dealing with severe uncertainty, the question arises: what exactly is the secret weapon that Ben-Haim's (2001, 2006) Info-Gap decision theory presumably provides for handling severe uncertainty? Specifically, what exactly is the hidden magic in

$$\hat{\alpha}(q, r_c) := \max\{\alpha : r_c \leq R(q, u), \forall u \in \mathcal{U}(\alpha, \tilde{u})\} \tag{10}$$

that supposedly enables Ben-Haim's (2001, 2006) Info-Gap decision theory to yield robust decisions under severe uncertainty?

As explained above, this is a rhetorical question the short answer to which is: There isn't any!

The long answer is a bit longer.

The non-probabilistic characteristic of this model is governed by the $\forall$ symbol. That is, the model assumes that Nature always selects the WORST u in $\mathcal{U}(\alpha, \tilde{u})$, where "worst" means " the least attractive element of $\mathcal{U}(\alpha, \tilde{u})$ as far as the performance requirement $r_c \leq R(q, u)$ is concerned".

In practical terms this means that in response to the decision maker's choice of a $q \in \mathcal{Q}$ and an $\alpha \geq 0$, Nature (namely Uncertainty) will always try to find a $u \in \mathcal{U}(\alpha, \tilde{u})$ that violates the performance requirement.

This, in turn, means that a rational decision maker will not choose a pair (q, α) such that some $u \in \mathcal{U}(\alpha, \tilde{u})$ violates the performance requirement $r_c \leq R(q, u)$.

In short, the robustness of a decision q is the largest value of α such that the performance constraint is satisfied for all $u \in \mathcal{U}(\alpha, \tilde{u})$. Hence,

$$\hat{\alpha}(q, r_c) := \max\{\alpha : r_c \leq R(q, u), \forall u \in \mathcal{U}(\alpha, \tilde{u})\} \ , \ q \in \mathcal{Q} \tag{11}$$

Talking about buzz words, in the parlance of classical decision theory this entire story is summarized by one word: **Maximin**.

The long story is as follows:

3.3 Maximin

This paradigm was formulated by the Hungarian born mathematician Abraham Wald (1902 - 1950), the founder of the field of statistical sequential analysis.

In plain language the recipe provided/dictated by this paradigm is as follows:

> **Maximin Maxim** (Wald 1945, 1950)
> Rank alternatives by their worst possible outcomes. That is, adopt the alternative the worst outcome of which is at least as good as the worst outcome of the others.

This is precisely what Ben-Haim's (2001, 2006) Info-Gap robustness model does: it ranks alternatives (values of α) based on the outcome (α) associated with the worst $u \in \mathcal{U}(\alpha, \tilde{u})$ with respect to the performance requirement. For a given value of q and a given value of α there are then only two types of u in $\mathcal{U}(\alpha, \tilde{u})$: a "good" u and "bad" u. The former satisfies the requirement $r_c \leq R(q, u)$ the latter does not.

In short, the best (optimal) α for a given decision q is the largest value of α such that the worst u in $\mathcal{U}(\alpha, \tilde{u})$ is "good".

In any case, the thing about the buzz word "Maximin" is that there is a substantial substance behind it. Indeed, Maximin is the prime tool of thought in classical decision theory (Resnik 1987, French 1988) and robust optimization (Kouvelis and Yu 1997, Ben-Tal et al 2006) .

Formally, Wald's (1945, 1950) Maximin model has two popular equivalent generic mathematical formulations, namely

Classical Format		Mathematical Programming Format
$\max_{a \in A} \min_{s \in S(a)} f(a, s)$	$=$	$\max_{\substack{a \in A \\ \alpha \in \mathbb{R}}} \{\alpha : \alpha \leq f(a, s), \forall s \in S(a)\}$

(12)

where

- A denotes the set of *alternatives* (decisions).
- $S(a)$ denotes the set of *states* associated with alternative a.
- $f(a, s)$ denotes the *outcome* (payoff) generated by a (a, s) pair.

In case you do not see it, Ben-Haim's (2001, 2006) Info-Gap robustness model is the instance of the generic Maximin model that is specified by the following constructs:

$$a = (q, \alpha) \tag{13}$$

$$s = u \tag{14}$$

$$A = \mathcal{Q} \times [0, \infty) \tag{15}$$

$$S(a) = \mathcal{U}(\alpha, \tilde{u}) \ , \ a = (q, \alpha) \tag{16}$$

$$f(a, s) = a\,(r_c \preceq R(q, s)) \ , \ a = (q, \alpha) \tag{17}$$

where

$$x \preceq y := \begin{cases} 1 & , \quad x \leq y \\ 0 & , \quad x > y \end{cases} \tag{18}$$

That is,

THEOREM (Sniedovich 2007):

$$\begin{array}{ccc} \text{Info-Gap Format} & & \text{Classical Maximin Format} \\ \hline \max\{\alpha : r_c \leq R(q,u), \forall u \in \mathcal{U}(\alpha,\tilde{u})\} & = & \max\limits_{\alpha\geq 0} \min\limits_{u\in\mathcal{U}(\alpha,\tilde{u})} \alpha\cdot(r_c \preceq R(q,u)) \end{array} \tag{19}$$

Proof. See (**??**).

In short, the good old buzz word "Maximin" can easily express the idea encapsulated in Ben-Haim's (2001, 2006) Info-Gap robustness model. It has been doing this kind of things – and much more – for more than 60 years.

Indeed, as indicated above, Wald's (1945, 1950) Maximin model is still one of the most important tools of thought in decision theory (Resnik 1987, French 1988) and robust optimization (Kouvelis and Yu 1997, Ben-Tal et al 2006).

3.4 Discussion

The following two comments on the `SlashDot` (news for nerds. Stuff that matters) website were triggered by a post entitled "AJAX Buzzword Reinvigorates Javascript".[4]

Given the low tech (household) meaning of the word `AJAX`, the first comment should have been anticipated:

> **AJAX also good for . . .** (Score:5, Funny)
> by Anonymous Coward on Tuesday May 24 2005, @12:19PM (#12624346)
>
> cleaning tub
> cleaning toilet
> getting first post

The second comment is more interesting and its last sentence is not only insightful but also very relevant to our discussion.

> **Are we sure it's the buzzword?** (Score:5, Insightful)
> by twifosp (532320) on Tuesday May 24 2005, @12:48PM (#12624671)
>
> I find it hard to believe that the buzzword itself breathed life back into Javascript like the title implies.
>
> I think maybe the slick apps like google maps is finally showing what good code CAN do, instead of the bloated bug ridden javascripting of yesterday.
>
> Or maybe I'm just not transcending expectations by thinking outside of the box, and therefore my toolset isn't capable of brigding the information gap causing a chasm with my ability to think forwardly.

[4]See `http://it.slashdot.org/article.pl?sid=05/05/24/159211&from=rss`.

> I'm struggling to identify which is worse: The day when we report that a buzzword has made progress, or the day a buzzword actually creates progress.

Buzz words have their role and place in decision theory. But they should be used with care, in moderation and for a purpose.

But the trouble with Ben-Haim's (2001, 2006) Info-Gap decision theory is not with the buzz words that it deploys. This is definitely not what my criticism of it is all about.

Rather, its failure to recognize the extent of its relationship with Maximin is the issue here. Instead of presenting its robustness model for what it is – a simple instance of Wald's (1945, 1950) Maximin model – this model is presented as something new that is radically different from all current models of decision-making under severe uncertainty. In fact, Ben-Haim (1999, 2005) actually claims that Info-Gap's robustness model is not a Maximin model!

But this is only part of the story. The other part is that for some inexplicable reason Ben-Haim's (2001, 2006) robustness model applies the Maximin maxim expressly in the immediate neighborhood of the estimate $\tilde{u}$. Under severe uncertainty this is unacceptable because the results generated by Ben-Haim's (2001, 2006) robustness model are only as good as the estimate on which they are based. Under conditions of severe uncertainty this estimate is a wild guess and therefore so are the results generated by Info-Gap's robustness model.

In short, in my view Ben-Haim's (2001, 2006) Info-Gap decision theory is a VOODOO DECISION THEORY. More specifically, it is a voodoo application of the Maximin maxim.

4 Conclusion

The real story behind Ben-Haim's (2001, 2006) Info-Gap decision theory is not the buzz word "information gap". The theory handles uncertainty in the usual Maximin manner. But its insistence on conducting this type of worst-case analysis expressly in the immediate neighborhood of a poor estimate renders this theory unsuitable for decision-making under severe uncertainty.

Yet, Ben-Haim's (2001, 2006) Info-Gap decision theory is presented and promoted exactly for this purpose: decision-making under severe uncertainty.

This fundamental flaw is precisely the reason behind my Maximin, Info-Gap, and Voodoo Decision Theory Campaigns (`www.moshe-online.com`).

Regarding Los and Tungsong (2008), as we have seen, it is a perfect example of the danger in off-the-cuff commentary on a concrete theory the title of which is buzzy.

Los and Tungsong (2008) are either not aware of the fact that Ben-Haim's (2001, 2006) Info-Gap decision theory is based on a specific robustness model, namely on

$$\hat{\alpha}(q, r_c) := \max\left\{\alpha \geq 0 : r_c \leq R(q,u), \forall u \in \mathcal{U}(\alpha, \tilde{u})\right\}\ ,\ q \in \mathcal{Q} \tag{20}$$

or, have failed to note that my criticism of Ben-Haim's (2001, 2006) Info-Gap decision theory refers implicitly and exclusively to . . . (surprise, surprise!) Ben-Haim's (2001, 2006) Info-Gap decision theory and to . . . no other theory.

Be it as it may, their commentary is misguided.

This episode vividly illustrates the point that progress in decision theory will not be achieved by deployment of buzz words and the re-invention of old wheels.

I examine Los and Tungsong (2008) specific comments on the Info-Gap/Maximin connection in the Appendix.

Appendix

A My criticism of Info-Gap decision theory

Info-Gap decision theory (Ben-Haim 2001, 2006) is a relatively young non-probabilistic theory that seeks robust decisions under conditions of SEVERE uncertainty. The official Info-Gap literature does not have any doubt as to Info-Gap's role and place in decision theory. Indeed, the message is crystal clear[5]:

> Info-gap decision theory is radically different from all current theories of decision under uncertainty. The difference originates in the modelling of uncertainty as an information gap rather than as a probability. The need for info-gap modeling and management of uncertainty arises in dealing with severe lack of information and highly unstructured uncertainty.
>
> Ben-Haim (2006, p. xii)

> In this book we concentrate on the fairly new concept of information-gap uncertainty, whose differences from more classical approaches to uncertainty are real and deep. Despite the power of classical decision theories, in many areas such as engineering, economics, management, medicine and public policy, a need has arisen for a different format for decisions based on severely uncertain evidence.
>
> Ben-Haim (2006, p. 11)

This is re-enforced by Info-Gap aficionados who seem to regard Info-Gap as a sort of *"breakthrough"* in decision theory and its contribution to decision theory as fundamental. For instance, you can find these assessments on the flyers[6] of the 2nd edition of the Info-Gap book (Ben-Haim, 2006):

> *Professor Yakov Ben-Haim has written a landmark book. ... His information-gap modeling approach to decision making under uncertainty constitutes a new and revolutionary approach for addressing tough decision problems when little information is available.*
>
> Prof. Keith Hipel, Dept. of Systems Design Engineering, University of Waterloo, Canada.

> *Ben-Haim's book is widely in demand by those in my field because of its revolutionary strategy implications.*
>
> Cliford C. Dacso, MD, MBA, Distinguished Research Professor, University of Houston, John S. Dunn Sr. Research Chair in General Internal Medicine.

The strange thing is that even the most superficial examination of Info-Gap decision theory reveals that its generic non-probabilistic robustness model is not

[5]Color is added in this page and elsewhere in this article for emphasis

[6]www.technion.ac.il/∼yakov/flyer02final.pdf , www.technion.ac.il/∼yakov/flyer01.pdf

radically different from models that have been used in the framework of classical decision theory for more than sixty years.

In fact, it is a simple exercise to show that this model is an instance of one the most famous models in classical decision theory, namely Wald's (1945, 1950) Maximin model.

Moreover, it is also a simple exercise to show that this specific instance of Wald's Maximin model is utterly unsuitable for the treatment of SEVERE uncertainty. This is so because the model is LOCAL in nature: it conducts the robustness analysis in the IMMEDIATE NEIGHBORHOOD of an ESTIMATE of the parameter of interest.

But, alas, according to Info-Gap's decision theory, under conditions of severe uncertainty the estimate is a wild guess, a poor indication of the true value of the parameter of interest and is likely to be substantially wrong. Thus, an application of the universal *GIGO*[7] *Axiom* warns us that the results generated by Info-Gap's robustness analysis are also wild guesses, of poor quality, and are likely to be substantially wrong.

So, in short, Info-Gap's robustness model does not tackle SEVERE uncertainty, it simply IGNORES it. This is surprising given the subtitle of the two Info-Gap books (Ben-Haim 2001, 2006): *decisions under severe uncertainty.*

In this appendix I quickly go, again, through a formal analysis of the two main[8] flaws in Info-Gap's decision theory:

· The misconception about its relationship with Wald's Maximin model.

· The local nature of Info-Gap's robustness model.

I shall then briefly explain the flaws in Los and Tungsong's (2008) assessment of the relationship between Ben-Haim's (2001, 2006) Info-Gap decision theory and Wald's (1945, 1950) Maximin model.

A.1 The decision problem

For the purposes of this discussion it is important to distinguish between the decision problem addressed by Ben-Haim's (2001, 2006) Info-Gap decision theory and the model that this theory deploys to formulate this problem formally.

In this section I examine the decision problem itself and in the next section I take a look at Ben-Haim's (2001, 2006) Info-Gap generic robustness model for this problem. I deliberately use Ben-Haim's (2001, 2006) notation for this purpose[9].

The generic problem under consideration is described by the following four simple objects:

· A *decision space* $\mathcal{Q}$.
This set contains all the decisions available to the decision maker.

· An *uncertainty space,* $\mathfrak{U}$.
This set represents the uncertainty in the true value of the parameter of interest. Let u denote a generic element of $\mathfrak{U}$. All we know is that one of the elements of $\mathfrak{U}$ is the true value of the parameter of interest, but we do not know which one.

[7] Garbage In - Garbage Out

[8] Details on other flaws can be found elsewhere. See my website moshe-online.com

[9] There is one exception: I use the symbol $\mathfrak{U}$ to denote the complete uncertainty space. Most of the Info-Gap articles do not refer to this set explicitly. In the two Info-Gap books (Ben-Haim 2001, 2006) the symbol S is used for this purpose.

- A *performance function* $R = R(q,u)$.
  Formally R is a real-valued function on $\mathcal{Q} \times \mathfrak{U}$.
- A *critical performance level*, r_c.
  This is a real number representing the minimum performance level that the performance function is required to achieve.

Our objective is to find the BEST decision, where

$$Best = most\ robust\ with\ respect\ to\ the\ performance\ constraint\ r_c \leq R(q,u). \tag{21}$$

Ideally then, we search for a decision $q' \in \mathcal{Q}$ that satisfies the performance requirement for all $u \in \mathfrak{U}$:

$$R(q',u) \geq r_c \ , \ \forall u \in \mathfrak{U} \tag{22}$$

We shall refer to such a decision as a *super-robust* decision and let $\mathcal{Q}^*$ denote the set of all the super-robust decisions, namely let

$$\mathcal{Q}^* := \{q \in \mathcal{Q} : r_c \leq R(q,u) \ , \ \forall u \in \mathfrak{U}\} \tag{23}$$

It is instructive to distinguish between the following three cases in relation to the cardinality of set $\mathcal{Q}^*$:

- $|\mathcal{Q}^*| = 0$: There are no super-robust decisions.
- $|\mathcal{Q}^*| = 1$: There exists exactly one super-robust decision.
- $|\mathcal{Q}^*| > 1$: There exists more than one super-robust decision.

Only the first case is of interest to us in this discussion, so henceforth we assume that $|\mathcal{Q}^*| = 0$. Regarding the other two cases:

- If $|\mathcal{Q}^*| = 1$ then the sole element of $\mathcal{Q}^*$ is the "best" decision.
- If $|\mathcal{Q}^*| > 1$ then secondary criteria are used to select the "best" decision in $\mathcal{Q}^*$.

Back to our case, namely $|\mathcal{Q}^*| = 0$.

The question is this: given that there are no super-robust decisions, what is a proper definition of robustness in this case? Given a pair of decisions, how do we determine which one is more robust?

Given that the uncertainty under consideration is SEVERE, it is intuitively obvious what kind of robustness the performance constraint under consideration, namely $r_c \leq R(q,u)$, calls for:

> A ROBUST decision is one that satisfies the performance requirement $r_c \leq R(q,u)$ over a LARGE region (subset) of the uncertainty space $\mathfrak{U}$.

With this in mind, let $\mathcal{V}(q)$ denote the subset of $\mathfrak{U}$ over which decision q satisfies the performance requirement, namely define

$$\mathcal{V}(q) := \{u \in \mathfrak{U} : r_c \leq R(q,u)\} \ , \ q \in \mathcal{Q} \tag{24}$$

Note that by definition $\mathcal{V}(q) \subseteq \mathfrak{U}, \forall q \in \mathcal{Q}$ and that $\mathcal{V}(q) = \mathfrak{U}$ for some $q \in \mathcal{Q}$ iff q is a super-robust decision.

In any case, for a decision q to be robust, $\mathcal{V}(q)$ should be a relatively large subset of the uncertainty space $\mathfrak{U}$.

So the decision problem under consideration can be stated more formally as follows:

> Find a decision $q \in \mathcal{Q}$ such that $\mathcal{V}(q) = \{u \in \mathfrak{U} : r_c \leq R(q,u)\}$ is as LARGE as possible.

However, strictly speaking this problem is not well defined because we have not yet defined how we measure the size of the sets $\mathcal{V}(q), q \in \mathcal{Q}$.

So let τ denote a real-valued function on the power set of the uncertainty space $\mathfrak{U}$ and interpret $\tau(U)$ as the SIZE of set $U \subset \mathfrak{U}$. The essential properties of τ are then as follows:

$$U' \subset U'' \rightarrow \tau(U') < \tau(U'') \tag{25}$$

$$\rho(\emptyset) = 0 \tag{26}$$

where $\emptyset$ denotes the empty set. Observe that this implies that $\tau(U) > 0, \forall U \subseteq \mathfrak{U}, U \neq \emptyset$.

In other words, we require $\tau(U)$ to be increasing with the "size" of set U, and therefore the size of a set should be larger than the size of any of its (proper) subsets.

We can regard the size of $\mathcal{V}(q)$, namely

$$\rho(q) := \tau(V(q)) = \tau\left(\{u \in \mathfrak{U} : r_c \leq R(q,u)\}\right) \ , \ q \in \mathcal{Q} \tag{27}$$

as the ROBUSTNESS of decision q.

The decision problem under consideration is then to find the most robust decision:

$$\rho^* := \max_{q \in \mathcal{Q}} \rho(q) \tag{28}$$

Note that in view of (25), it follows from (27) that

$$\rho(q) := \tau\left(\{u \in \mathfrak{U} : r_c \leq R(q,u)\}\right) \tag{29}$$

$$= \sup_{U \subseteq \mathfrak{U}} \{\tau(U) : r_c \leq R(q,u), \forall u \in U\} \tag{30}$$

In words, the best decision is one that satisfies the performance requirement $r_c \leq R(q,u)$ over the LARGEST subset of the uncertainty set $\mathfrak{U}$.

The appearance of sup, rather than max, in this expression is a reflection of the fact that we have not imposed any conditions on the argument of τ, neither did we require τ to be continuous. Hence, there is no guarantee that τ attains a maximum value on the power set of $\mathfrak{U}$.

So, to rid the discussion of technical diversions of this kind, consider this:

ASSUMPTION:

$$\sup_{U \subseteq \mathfrak{U}} \{\tau(U) : r_c \leq R(q,u), \forall u \in U\} = \max_{U \subseteq \mathfrak{U}} \{\tau(U) : r_c \leq R(q,u), \forall u \in U\} \tag{31}$$

for all $q \in \mathcal{Q}$.

For instance, this will be the case when $|\mathfrak{U}| < \infty$, namely in cases where the uncertainty space $\mathfrak{U}$ consists of finitely many elements.

In any case, under this assumption, the robustness of a decision is as follows:

$$\rho(q) := \tau(\mathcal{V}(q)) = \max_{U \subseteq \mathfrak{U}} \{\tau(U) : r_c \leq R(q,u), \forall u \in U\} \ , \ q \in \mathcal{Q} \tag{32}$$

In words,

> The ROBUSTNESS of a decision is the "size" of the largest region of uncertainty over which the decision satisfies the performance requirement.

So as we can clearly see, the crux of the matter is to find a meaningful and useful formulation (definition) for the "size" of a region of uncertainty, namely the size of a subset of the uncertainty space $\mathfrak{U}$.

Comment: Needless to say, the definition of "size" – hence its interpretation as robustness – must be handled with care and must be, in one way or another, compared with the "size" of the uncertainty space $\mathfrak{U}$. Indeed, in the framework of the preceding discussion robustness is a "relative" notion: we ask ourselves how large the "safe" part of $\mathfrak{U}$ is, that is the part where the performance requirement is satisfied. This part can be very large in some absolute sense, but very small relative to the uncertainty space $\mathfrak{U}$, and vice versa.

★

It should also be pointed out that in the above formulation the robustness of a decision is regarded as a *global* property in that a priori we do not confine the analysis to any particular region of the uncertainty space $\mathfrak{U}$ under consideration. This is a reflection of the fact that in this discussion we are dealing with decision-making under SEVERE uncertainty. In this environment there is no reason to restrict the analysis to any particular subset of $\mathfrak{U}$. We therefore ask ourselves:

> What is the LARGEST subset of the uncertainty space over which the performance requirement can be satisfied?

So far so good.

A.2 Info-Gap's robustness model

In contrast to the GLOBAL attitude towards robustness that we took in the discussion so far, Ben-Haim's (2001, 2006) Info-Gap decision theory adopts a very LOCAL approach. For this reason a key role in Ben-Haim's (2001, 2006) Info-Gap robustness model is played by an ESTIMATE of the true value of the parameter of interest $u \in \mathfrak{U}$.

In other words, Ben-Haim's (2001, 2006) Info-Gap decision theory assumes that we have in our possession an estimate, call it $\tilde{u}$, of the true value of the parameter of interest, and that robustness is a LOCAL property measured in the IMMEDIATE NEIGHBORHOOD of this estimate.

So much so, that Ben-Haim's (2001, 2006) Info-Gap regions of uncertainty are *nested sets centered at the estimate* $\tilde{u}$. More formally, Ben-Haim's (2001, 2006)

Info-Gap robustness model deploys a family of sets $\mathcal{U}(\alpha,\tilde{u}) \subseteq \mathfrak{U}, \alpha \geq 0$ – centered at $\tilde{u}$ – with the NESTING property

$$\mathcal{U}(\alpha,\tilde{u}) \subseteq \mathcal{U}(\alpha+\varepsilon,\tilde{u}) \ , \ \forall \alpha, \varepsilon > 0 \tag{33}$$

$$\mathcal{U}(0,\tilde{u}) = \{\tilde{u}\} \tag{34}$$

The parameter α represents the "size" of the region $\mathcal{U}(\alpha,\tilde{u})$ and is interpreted as the "horizon of uncertainty". A schematic layout of these regions is shown in Figure 1. The concentric circles represent the uncertainty regions whose center point represents the estimate $\tilde{u}$. The radius of a circle represents the "size", α, of the region of uncertainty.

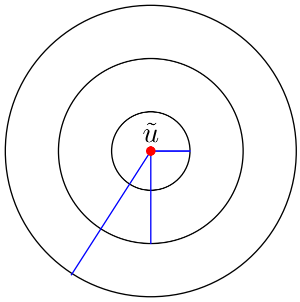


Figure 1: Info-Gap regions of uncertainty, $\mathcal{U}(\alpha,\tilde{u}), \alpha \geq 0$

For simplicity we assume that the performance constraint $r_c \leq R(q,\tilde{u})$ is satisfied by all decisions $q \in \mathcal{Q}$ at $u = \tilde{u}$. This is just a modeling technicality: if $r_c > R(q,\tilde{u})$ for some decision $q \in \mathcal{Q}$ then this decision can be discarded at the outset.

So in this framework the robustness of decision q is the largest value of α such that the performance requirement is satisfied for all u in $\mathcal{U}(\alpha,\tilde{u})$. More formally, the robustness of decision q is defined as follows:

$$\hat{\alpha}(q,r_c) := \max \ \{\alpha : r_c \leq R(q,u), \forall u \in \mathcal{U}(\alpha,\tilde{u})\} \ , \ q \in \mathcal{Q} \tag{35}$$

Consequently, the "best" decision is one whose robustness, namely $\hat{\alpha}(q,r_c)$ value, is the largest.

To appreciate the difficulties with the local nature of Ben-Haim's (2001, 2006) Info-Gap robustness model, consider the case where $\mathfrak{U} = \mathbb{R}$, $\tilde{u} = 0$, $r_c = 0$ and

$$\mathcal{U}(\alpha,\tilde{u}) : = [-\alpha,\alpha] \ , \ \alpha \geq 0 \tag{36}$$

$$R(q',u) : = 1 + 6u^2 - 6|u| \ , \ u \in \mathbb{R} := (-\infty,\infty) \tag{37}$$

for some decision $q' \in \mathcal{Q}$.

Then according to Ben-Haim's (2001, 2006) Info-Gap robustness model the robustness of this decision is as follows:

$$\hat{\alpha}(q',r_c) : = \max \ \{\alpha : r_c \leq R(q',u), \forall u \in \mathcal{U}(\alpha,\tilde{u})\} \tag{38}$$

$$= \max \ \{\alpha : 0 \leq 1 + 6u^2 - 6|u|, \forall u \in [-\alpha,\alpha]\} \tag{39}$$

$$= 0.21132486540519 \tag{40}$$

Note that in this example the uncertainty space is the entire real line $\mathbb{R}$ and that decision q' satisfies the performance requirement everywhere on this space except on the very small intervals

$$\Delta^{+} = [0.21132486540519, 0.7886751345981] \tag{41}$$

$$\Delta^{-} = [-0.7886751345981, -0.21132486540519] \tag{42}$$

This is shown graphically in Figure 2.

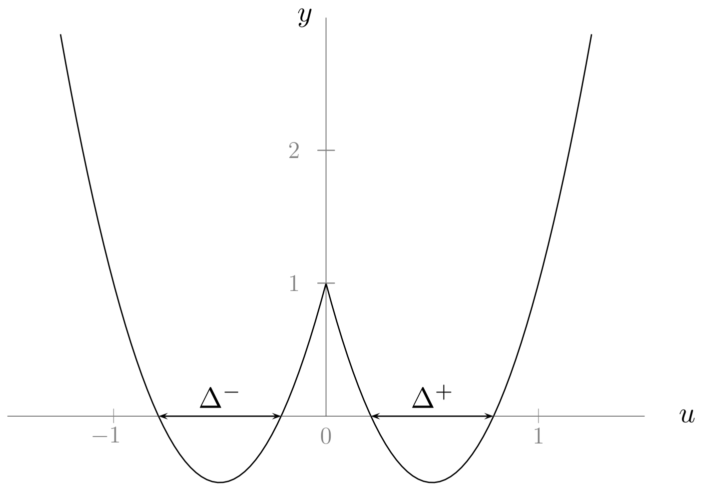


Figure 2: $y = 1 + 6u^2 - 6|u|$

So clearly, globally, that is over $\mathfrak{U} = (-\infty, \infty)$, this decision is extremely robust. But according to Ben-Haim's (2001, 2006) Info-Gap robustness model this decision is not robust at all: it is "safe" only on the very small interval $[-\delta, \delta], \delta = 0.21132486540519$.

The local nature of Ben-Haim's (2001, 2006) Info-Gap's robustness model manifests itself more clearly when we compare the robustness of two decisions. So, for example, consider another decision, say $q'' \in \mathcal{Q}$, for which

$$R(q'', u) := 0.2 - 4x^2 \ , \ u \in \mathbb{R} := (-\infty, \infty) \tag{43}$$

Then according to Ben-Haim's (2001, 2006) Info-Gap robustness model the robustness of this decision is as follows:

$$\hat{\alpha}(q'', r_c) := \max \{\alpha : r_c \leq R(q'', u), \forall u \in \mathcal{U}(\alpha, \tilde{u})\} \tag{44}$$

$$= \max \{\alpha : 0 \leq 0.2 - 4u^2, \forall u \in [-\alpha, \alpha]\} \tag{45}$$

$$= \sqrt{0.05} = 0.22360679774998 \tag{46}$$

Since $\hat{\alpha}(q'', r_c) > \hat{\alpha}(q', r_c)$, it follows that – according to Ben-Haim's (2001, 2006) Info-Gap decision theory – decision q'' is more robust then decision q'.

This does not make much sense:

- q' is robust on almost the entire region of uncertainty.
- q'' is robust only on a minute subinterval of the entire region of uncertainty.

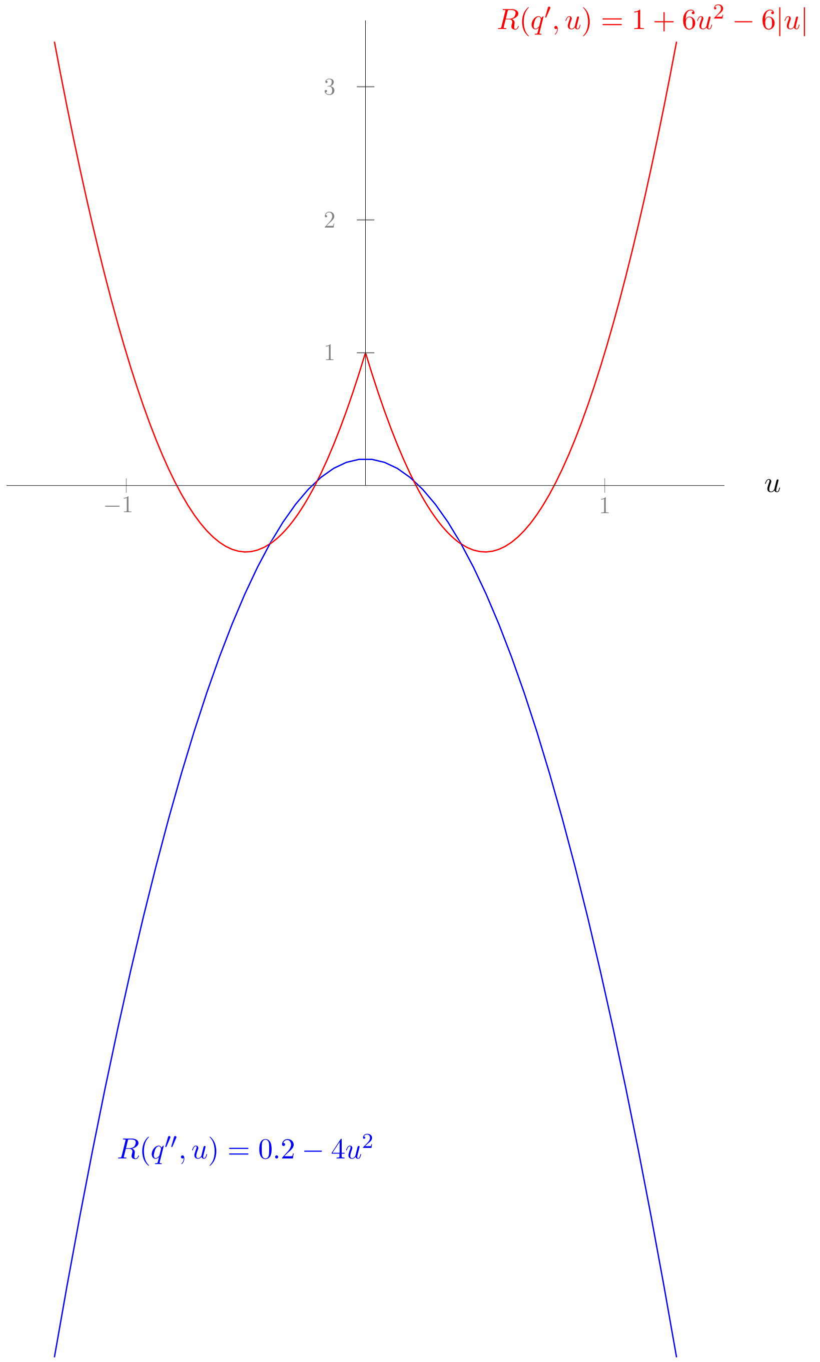


Figure 3: A comparison of $R(q', u) = 1 + 6u^2 - 6|u|$ and $R(q'', u) = 0.2 - 4u^2$

· Yet Ben-Haim's (2001, 2006) Info-Gap decision theory claims that q'' is more robust than q'.

Figure 3 speaks for itself.

This is a manifestation of the fact that by definition Ben-Haim (2001, 2006) regards robustness as a local property exhibiting the behavior of the performance function in the immediate neighborhood of the estimate.

Recall that the robustness of a decision á la Info-Gap is the size of the largest region of uncertainty over which the decision satisfies the performance requirement at all points in the region. But there is a fine print to this description. It reads as follows:

Warning

Info-Gap's regions of uncertainties, $\mathcal{U}(\alpha, \tilde{u}), \alpha \geq 0$, are nested and are all centered at $\tilde{u}$. Therefore, by definition, robustness á la Info-Gap is a local property that does not necessarily represent robustness over the entire region of uncertainty $\mathfrak{U}$.
For this reason Info-Gap's robustness model is unsuitable for decision-making under severe uncertainty. It does not tackle the severity of the uncertainty, it simply ignores it.

Comment: Ben-Haim's (2006) Info-Gap decision theory correctly argues that under conditions of SEVERE uncertainty the estimate $\tilde{u}$ is a wild guess, a poor indication of the true value of the u and can be substantially wrong.

It is therefore astonishing that the same theory deploys a local robustness model that conducts the robustness analysis solely in the immediate neighborhood of this estimate.

This is the reason why I regard Ben-Haim's (2001, 2006) Info-Gap decision theory as a VOODOO DECISION THEORY par excellence. A formal critique of Ben-Haim's (2001, 2006) Info-Gap robustness model can be found on my website and on WIKIPEDIA[10]. I briefly discuss a related flaw of Ben-Haim's (2001, 2006) Info-Gap robustness model in §A.4. ★

A.3 The Minimax connection

The Info-Gap literature is emphatic that Ben-Haim's (2001, 2006) Info-Gap robustness model is not a Maximin model. For example, Ben-Haim (1999, pp. 271-2) argues as follows:

> We note that robust reliability is emphatically not a worst-case analysis. In classical worst-case min-max analysis the designer minimizes the impact of the maximally damaging case. But an info-gap model of uncertainty is an unbounded family of nested sets: $U(\alpha, \tilde{u})$, for all $\alpha \geq 0$. Consequently, there is no worst case: any adverse occurrence is less damaging than some other more extreme event occurring at a larger value of α. What Eq. (1) expresses is the greatest level of uncertainty consistent with no-failure. When the designer chooses q to maximize $\hat{\alpha}(q, r_c)$ he is maximizing his immunity to an unbounded ambient uncertainty. The

[10]See www.wikipedia.com/wiki/info-gap_decision_theory

closest this comes to "min-maxing" is that the design is chosen so that "bad" events (causing reward R less than r_c) occur as "far away" as possible (beyond a maximized value of $\hat{\alpha}$).

As I repeatedly explained on many occasions and in many articles, such statements betray a serious misconception about worst-case analysis and the type of worst-case analysis that Ben-Haim's (2001, 2006) Info-Gap robustness analysis is conducting. I give it another go here, so here it comes.

Ben-Haim (1999, pp. 271-2, and in other articles) confuses two distinct worst-case analyses:

- Worst case analysis of $R(q,u)$ over $u \in \mathfrak{U}$.
- Worst-case analysis of the performance requirement $r_c \leq R(q,u)$ over $\mathcal{U}(\alpha,\tilde{U})$ for a **given** value of α.

The issue as to whether there is a worst case for $R(q,u)$ over $u \in \mathfrak{U}$ is not on the agenda in the framework of Ben-Haim's (2001, 2006) Info-Gap robustness analysis. That is, Ben-Haim's (2001, 2006) Info-Gap robustness analysis is not interested at all in the value of $R(q,u)$ per se: all that it is interested in is whether the performance constraint $r_c \leq R(q,u)$ is satisfied.

Differently put: Ben-Haim's (2001, 2006) Info-Gap robustness is not about $R(q,u)$ as such, it is about the constraint $r_c \leq R(q,u)$.

Thus, strictly speaking the argument that $R(q,u)$ does not have a worst case over $u \in \mathfrak{U}$ is not relevant at all to our discussion. Nevertheless, I must say something about this idea here because ... it is so wrong from a technical point of view. But in order not to disturb the flow of the discussion, I discuss this point in §A.8.

Now, by inspection, Info-Gap's robustness model (Ben-Haim 2001, 2006), namely

$$\hat{\alpha}(q,r_c) := \max\ \{\alpha : r_c \leq R(q,u), \forall u \in \mathcal{U}(\alpha,\tilde{u})\}\ ,\ q \in \mathcal{Q} \tag{47}$$

is precisely a worst-case analysis of the latter type: the $\forall$ requirement insists that whether or not a given value of α is "safe" with respect to a given decision q is determined by the worst u in $\mathcal{U}(\alpha,\tilde{u})$ as far as the constraint $r_c \leq R(q,u)$ is concerned.

Indeed, the existence of such a worst case is precisely what prevents the robustness from increasing indefinitely. In particular, consider some arbitrary decision, call it q', and assume that $\alpha' := \hat{\alpha}(q',r_c) < \infty$.

Since Ben-Haim's (2001, 2006) Info-Gap regions of uncertainty are nested, it follows that for any $\varepsilon > 0$ there exists a $u' \in \mathcal{U}(\alpha' + \varepsilon, \tilde{u})$ such that $R(q',u') < r_c$. In other words, $\mathcal{U}(\alpha' + \varepsilon, \tilde{u})$ contains a worst-case value of u: this value violates the performance requirement $r_c \leq R(q',u)$, and there is no more damaging u in the uncertainty space.

Ben-Haim's (2001, 2006) Info-Gap robustness model does not distinguish between various levels of violations of the performance constraint: the constraint is either satisfied or violated. If $r_c > R(q',u)$ for some $u \in \mathcal{U}(\alpha,\tilde{u})$ then – according to Ben-Haim's (2001, 2006) Info-Gap's robustness model – α is too large.

This simple observation provides the modeling "hint" as to how we should go about formulating Ben-Haim's (2001, 2006) Info-Gap's robustness model as a Maximin model (Wald 1945, 1950) . That is, consider the "penality" function

$$f(q,\alpha,u) := \begin{cases} \alpha & , \quad r_c \leq R(q,u) \\ -\infty & , \quad r_c > R(q,u) \end{cases}\ ,\ q \in \mathcal{Q}, \alpha \geq 0, u \in \mathcal{U}(\alpha,\tilde{u}) \tag{48}$$

By construction, if $q \in \mathcal{Q}, \alpha \geq 0, u \in \mathcal{U}(\alpha, \tilde{u})$ then

$$r_c \leq R(q,u) \longleftrightarrow f(q,\alpha,u) = \alpha \tag{49}$$

$$r_c > R(q,u) \longleftrightarrow f(q,\alpha,u) = -\infty \tag{50}$$

Thus, the huge penalty imposed by this function if we violate the performance constraint will insure that we do not increase the value of α to allow a $u \in \mathcal{U}(\alpha, \tilde{u})$ to violate the performance constraint.

In short,

MAXIMIN THEOREM:

Ben-Haim's (2001, 2006) Info-Gap robustness models is a simple instance of Wald's Maximin model. Specifically,

$$\hat{\alpha}(q,r_c) = \overbrace{\max\{\alpha : r_c \leq R(q,u), \forall u \in \mathcal{U}(\alpha,\tilde{u})\}}^{\text{Info-Gap Robustness Model}} = \overbrace{\max_{\alpha \geq 0} \min_{u \in \mathcal{U}(\alpha,\tilde{u})} f(q,\alpha,u)}^{\text{Classical Maximin Format}} \tag{51}$$

Proof.

$$\overbrace{\max_{\alpha \geq 0} \min_{u \in \mathcal{U}(\alpha,\tilde{u})} f(q,\alpha,u)}^{\text{Classical Maximin Format}} = \overbrace{\max_{\substack{\alpha \geq 0 \\ z \in \mathbb{R}}} \{z : z \leq f(q,\alpha,u), \forall u \in \mathcal{U}(\alpha,\tilde{u})\}}^{\text{Mathematical Programming Format}} \tag{52}$$

$$= \max_{\alpha \geq 0} \{\alpha : \alpha \leq f(q,\alpha,u), \forall u \in \mathcal{U}(\alpha,\tilde{u})\} \tag{53}$$

$$= \max_{\alpha \geq 0} \{\alpha : r_c \leq R(q,u), \forall u \in \mathcal{U}(\alpha,\tilde{u})\} \tag{54}$$

$$= \overbrace{\max \{\alpha \geq 0 : r_c \leq R(q,u), \forall u \in \mathcal{U}(\alpha,\tilde{u})\}}^{\text{Info-Gap Robustness Model}} \tag{55}$$

★

As clearly indicated by this simple Maximin model (Wald 1945, 1950), Info-Gap's robustness model (Ben-Haim 2001, 2006) conducts its worst-case analysis over each of the regions of uncertainty $\mathcal{U}(\alpha, \tilde{u}), \alpha \geq 0$, one-at-a-time so to speak, rather then on the entire uncertainty space $\mathfrak{U}$.

However, if you are allergic to doing the worst-case analysis over the sets $\mathcal{U}(\alpha, \tilde{u}), \alpha \geq 0$, and prefer to conduct it always over the entire uncertainty space $\mathfrak{U}$, simple let

$$g(q,\alpha,u) := \begin{cases} \alpha & , \quad r_c \leq R(q,u), u \in \mathcal{U}(\alpha,\tilde{u}) \\ -\infty & , \quad \text{otherwise} \end{cases} \quad , \ q \in \mathcal{Q}, \alpha \geq 0, u \in \mathfrak{U} \tag{56}$$

in which case

MAXIMIN COROLLARY: for any $q \in \mathcal{Q}$ we have

$$\hat{\alpha}(q,r_c) = \max\{\alpha : r_c \leq R(q,u), \forall u \in \mathcal{U}(\alpha,\tilde{u})\} = \max_{\alpha \geq 0} \min_{u \in \mathfrak{U}} g(q,\alpha,u) \tag{57}$$

In short, Ben-Haim's (2001, 2006) Info-Gap's robustness model is a simple Maximin model (Wald 1945, 1950). More on this fact can be found on my website and on WIKIPEDIA.

A.4 The (Mis)-Treatment of severe uncertainty

As indicated by the subtitle[11] of the two Info-Gap books (Ben-Haim, 2001, 2006), and as repeatedly emphasized in the official Info-Gap literature, Ben-Haim's (2001, 2006) Info-Gap decision theory is supposed to tackle SEVERE uncertainty.

But as I have repeatedly shown in my articles and presentations, Info-Gap is doing the exact opposite. In other words, Ben-Haim's (2001, 2006) Info-Gap robustness model simply IGNORES the severity of the uncertainty it is supposed to tackle.

This is so because Ben-Haim's (2001, 2006) Info-Gap robustness model is LOCAL in nature: it examines only the IMMEDIATE NEIGHBORHOOD of the given estimate of the parameter of interest. As a result, in principle, Ben-Haim's (2001, 2006) Info-Gap robustness model is utterly invariant to changes in the severity of the uncertainty under consideration.

Formally, this fundamental flaw in Ben-Haim's (2001, 2006) Info-Gap decision theory can be stated in terms of the severe insensitivity of Ben-Haim's (2001, 2006) Info-Gap robustness model to the "size" of the uncertainty space $\mathfrak{U}$:

> INVARIANCE THEOREM (Sniedovich 2007):
> The robustness of a decision is invariant with the uncertainty space $\mathfrak{U}$. More specifically, let q^* be an arbitrary decision, set $\alpha^* = \hat{\alpha}(q^*, r_c) + \varepsilon$, where ε is any strictly positive number (can be arbitrarily small, but positive).
>
> Then the robustness of q^* is invariant with $\mathfrak{U}$ for all $\mathfrak{U}$ such that
>
> $$\mathcal{U}(\alpha^*, \tilde{u}) \subseteq \mathfrak{U} \tag{58}$$
>
> **Proof.** Directly from the nesting property of Info-Gap's regions of uncertainty. That is, from the definition of $\hat{\alpha}(q^*, r_c)$ it follows that for any $\varepsilon > 0$ there is a $u^* \in \mathcal{U}(\hat{\alpha}(q^*, r_c), \tilde{u})$ such that $r_c > R(q^*, u^*)$. The nesting property implies then that $u^* \in \mathcal{U}(\hat{\alpha}(q^*, r_c) + \varepsilon, \tilde{u})$ for all $\alpha \geq 0$. This in turn implies that $\hat{\alpha}(q^*, r_c)$ is independent of $\mathfrak{U}$ as long as $\mathcal{U}(\alpha^*, \tilde{u}) \subseteq \mathfrak{U}$.
> ★

This invariance property of Ben-Haim (2001, 2006) Info-Gap robustness model is illustrated graphically in Figure 4. Here, the robustness of decision q^* remains unchanged regardless by how much we increase the uncertainty space: it is the same for any set $\mathfrak{U}$ containing $\mathcal{U}(\alpha^*, \tilde{u})$.

In this particular illustration, the robustness of q^* remains the same as we increase the uncertainty space from $\mathfrak{U}'$, to $\mathfrak{U}''$, to $\mathfrak{U}'''$ and to $\mathfrak{U}''''$. In fact, the robustness of q^* will remain the same as long as $\mathfrak{U}$ contains $\mathcal{U}(\alpha^*, \tilde{u})$.

This is truly incredible!

Here we have a decision theory that claims to deal specifically with SEVERE uncertainty, yet its robustness model is so utterly insensitive to the severity of the uncertainty, measured by the "size" of the uncertainty space. Indeed, this insensitivity is what makes Info-Gap decision theory (Ben-Haim 2001, 2006) a VOODOO DECISION THEORY par excellence:

[11] Decisions Under Severe Uncertainty

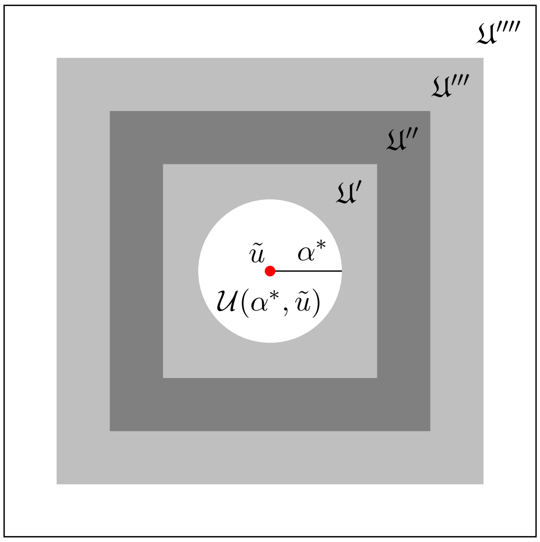


Figure 4: Invariance property, $\alpha^* = \hat{\alpha}(q^*, r_c) + \varepsilon$

Severe uncertainty ?!?!?
No worries, mate!
A wild guess and its immediate neighborhood will do!!

If we accept this recipe then how would we distinguish SEVERE uncertainty from a VERY MILD uncertainty where the estimate we have is a very good – but not perfect – indication of the true value of the parameter of interest?

The fact of the matter is that Info-Gap decision theory (Ben-Haim 1996, 2001, 2006) does not distinguish between very mild uncertainty and severe uncertainty.

A.5 The "How wrong can the model and data be?" myth

One finds in the Info-Gap literature numerous claims that Info-Gap theory is capable of answering questions such as these:

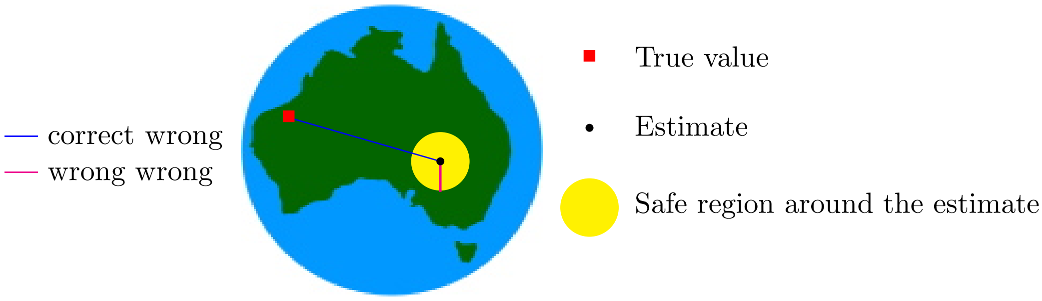
> *How wrong can the model and data be without jeopardizing the quality of the outcome?*
>
> *How wrong can this model be before I should change my decision?*

Such claims represent a serious misconception of Info-Gap's definition of robustness. To begin with, there is no way of knowing how wrong the model and data are because the true value of the parameter of interest is unknown – indeed is subject to *severe* uncertainty. And, Info-Gap's robustness is evaluated only in the *immediate neighborhood of this poor estimate.*

The correct interpretation of Info-Gap robustness is as follows:

> *The robustness of a decision is the maximum deviation from a* *given estimate* *such that the performance requirement is satisfied for every value of the parameter in the immediate neighborhood of the estimate stipulated by this deviation.*

That is, by definition Info-Gap's robustness is an inherently local property. The picture is this:



In other words, all that Info-Gap's robustness tells us is how "safe" we are in the immediate neighborhood of the estimate.

The trouble is, of course, that subject to *severe* uncertainty the estimate is a wild *guess,* . . . a *poor* . . . substantially *wrong,* . . . etc, etc, etc.

As I indicated already, it is a simple exercise to construct examples where a decision is highly robust in the neighborhood of the estimate, but fragile elsewhere in the total region of uncertainty, and vice versa.

In any case, it is really amazing how many analysts mistakenly interpret the definition of Info-Gap's robustness as being global while in fact it is crystal clear that it is local.

A.6 Voodoo decision theories

In my criticism of Ben-Haim's (2001, 2006) Info-Gap decision theory (eg. this paper) I have been making frequent references to Voodoo Decision Theory. In this section I explain what I mean by this and why I consider Ben-Haim's (2001, 2006) Info-Gap decision theory a classical voodoo decision theory.

According to Encarta online Encyclopedia

Voodoo n

1. A religion practiced throughout Caribbean countries, especially Haiti, that is a combination of Roman Catholic rituals and animistic beliefs of Dahomean enslaved laborers, involving magic communication with ancestors.
2. Somebody who practices voodoo.
3. A charm, spell, or fetish regarded by those who practice voodoo as having magical powers.
4. A belief, theory, or method that lacks sufficient evidence or proof.

The last meaning applies here:

> A voodoo decision theory is a decision theory that lacks sufficient evidence or proof.

The question is then: what kind of evidence or proof should we use to certify a decision theory to be non-Voodoo?

This is a difficult question.

There are no agreed upon criteria for such proofs and evidences and therefore it is not clear how this issue can be resolved.

However, in the case of Ben-Haim's (2001, 2006) Info-Gap decision theory the situation is quite simple because we can apply the universal GIGO AXIOM,[12] namely

GIGO Axiom

Garbage In ⟶ Model ⟶ Garbage Out

Wrong ⟶ Model ⟶ Wrong

Wild Guess ⟶ Model ⟶ Wild Guess

In fact, we can be more specific here:

> The robustness of any decision and the risk incurred in making that decision is only as good as the estimates on which it is based. Making estimation even more challenging, virtually all estimates that affect decisions are uncertain. Uncertainty can not be eliminated, but it can be managed.
>
> Top Ten Challenges for Making Robust Decisions
> The Decision Expert Newsletter, Volume 1; Issue 2
> `http://www.robustdecisions.com/newsletter0102.php`

Using this convention we conclude that the results generated by a robustness model are only as good as the estimates on which they are based. This suggests the following:

SUFFICIENT CONDITION:
Any decision theory whatsoever that knowingly does not subscribe to the GIGO Axiom is a Voodoo Decision Theory.

THEOREM
Ben-Haim's (2001, 2006) Info-Gap decision theory is a Voodoo Decision Theory.

Proof.

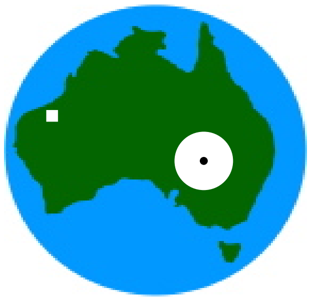
- Ben-Haim's (2001, 2006) Info-Gap decision theory is designed specifically for decision-making under severe uncertainty.
- Info-Gap theory is fully aware of the fact that under these conditions the estimate $\tilde{u}$ is a wild guess and is likely to be substantially wrong.
- Thus, Info-Gap's robustness model knowingly confines its analysis to the immediate neighborhood of a poor estimate that is likely to be substantially wrong.

[12] Garbage In – Garbage Out.

- Thus, Info-Gap decision theory does not subscribe to the GIGO Axiom.
- Hence, Info-Gap decision theory is a Voodoo Decision Theory. ★

In a nutshell, robustness à la info-gap is defined in total disregard for the *severity* of the uncertainty. I usually describe this debacle as a *Treasure Hunt*. The picture is this:

Treasure Hunt



- The island represents the region of uncertainty under consideration (the region where the treasure is located).
- The tiny black dot represents the estimate of the parameter of interest (estimate of the location of the treasure).
- The large white circle represents the region of uncertainty affecting info-gap's robustness analysis.
- The small white square represents the true (unknown) value of the parameter of interest (true location of the treasure).

Hence, info-gap may conduct its robustness analysis in the vicinity of Brisbane (QLD), whereas for all we know the true location of the treasure may be somewhere in the middle of the Simpson desert (AUS) or perhaps in down town Melbourne (VIC). Perhaps.

A.7 The "That's the best we have!" syndrome

Proponents of Info-Gap decision theory do not refute my criticism as such, they just do not agree with my ... conclusions.

There seems to be a sort of an agreement on the following points:

- Under severe uncertainty we have to assume that the estimate is a wild guess and is likely to be substantially wrong.
- Info-Gap's robustness is, by definition, local in nature in that it evaluates resilience to change in the neighborhood of an estimate.
- Info-Gap's robustness does not tell us much the robustness of decisions over the entire region of uncertainty.
- Actually, Info-Gap decision theory does not tackle the severity of the uncertainty under consideration.

My conclusion is that the above means that

- Info-Gap decision theory is fundamentally flawed.
- Info-Gap decision theory is unsuitable for robust decision-making under severe uncertainty

Let me explain:

> Given that Info-Gap decision theory presents itself, and is promoted, as a theory designed specifically for decisions under severe uncertainty, it is obvious that it is flawed and that it fails to deliver the goods.
>
> But it is more than that. The flaw is fundamental. To wit:

> By localizing the robustness analysis to the immediate neighborhood of an estimate that is known to be a wild guess and is likely to be substantially wrong, Info-Gap decision theory demonstrates complete disrespect for what the GIGO Axiom stands for and what it represents.

In other words, Ben-Haim's (2001, 2006) emphasizes the challenges posed by severe uncertainty – yet completely ignores these very challenges in the formulation of the robustness model. Indeed, if you care to probe this matter more closely you would compare the first book on Info-Gap, entitled *Robust reliability in the Mechanical Sciences* (Ben-Haim 1996), to the two more recent ones (Ben-Haim 2001, 2006), and you will discover that the notion *severe* uncertainty – now the fulcrum of info-gap – is something of an afterthought. In other words, you will discover that in the 1996 book no mention whatsoever is made of the uncertainty under consideration being *severe.* To the contrary, the overall impression is that the uncertainty is very mild.

Yet, precisely the same model, employing precisely the same treatment of uncertainty, is transferred lock stock and barrel to the two more recent info-gap books (Ben-Haim, 2001, 2006) whose concern is with decision under *severe* uncertainty. So, inexplicably, the selfsame methodology suddenly becomes a methodology for dealing with *severe,* in fact *Knightian*, uncertainty.

But Info-Gap proponents are not impressed at all by these arguments. They counter my criticism by the following

That's the best estimate we have!

My reply is that the fundamental flaw in Info-Gap decision theory is not that it utilizes an inferior estimate. Under severe uncertainty the best estimate we have is still a wild guess and is likely to be substantially wrong. This is a fact of life which we have to accept.

The fundamental flaw in Info-Gap decision theory is that it does not do anything about the severity of the uncertainty. This, I remind my Info-Gap colleagues, is in sharp contrast to the manner in which severe uncertainty is managed by ROBUST OPTIMIZATION METHODS (Kouvelis and Yu 1997, Ben-Tal et al 2006) and ROBUST DECISION-MAKING (Lempert et al 2006).

But Info-Gap proponents are not convinced by this argument:

> However, a major purpose of decision analysis is to provide focus for subjective judgments. That is, regardless of the formal analysis, a framework for discussion is provided. Without entering into any particular framework, or characteristics of frameworks in general, discussion follows about proposals for such frameworks.
>
> WIKIPEDIA
> `http://en.wikipedia.org/wiki/Info-gap_decision_theory`

I have to admit that I find it extremely difficult to figure out the meaning of the last two sentences in this quote. I suspect that they argue that my criticism does not mean that Info-Gap decision theory is completely useless. For, regardless of its flaws, the theory does provides a framework for discussion.

Indeed, I had a number of interesting discussions on the degree of uselessness of Info-Gap decision theory.

However, I leave it to the reader to decide on this issue, recalling that Info-Gap decision theory is supposed to be a theory that seeks robust decisions under conditions of severe uncertainty – actually Knightian uncertainty.

A.8 Worst-case analysis over unbounded regions

Ben-Haim (eg. 1999, 2005) persistently argues that Info-Gap's robustness model is not a Maximin model and that robustness á la Info-Gap is not a worst-case analysis. For example,

> We note that robust reliability is emphatically not a worst-case analysis. In classical worst-case min-max analysis the designer minimizes the impact of the maximally damaging case. But an info-gap model of uncertainty is an unbounded family of nested sets: $U(\alpha, \tilde{u})$, for all $\alpha \geq 0$. Consequently, there is no worst case: any adverse occurrence is less damaging than some other more extreme event occurring at a larger value of α. What Eq. (1) expresses is the greatest level of uncertainty consistent with no-failure. When the designer chooses q to maximize $\hat{\alpha}(q, r_c)$ he is maximizing his immunity to an unbounded ambient uncertainty. The closest this comes to "min-maxing" is that the design is chosen so that "bad" events (causing reward R less than r_c) occur as "far away" as possible (beyond a maximized value of $\hat{\alpha}$).
>
> Ben-Haim (1999, pp. 271-2)

Before we consider Info-Gap's robustness model, let us immediately dispel the notion that Maximin analysis cannot be conducted on unbounded sets.

Consider the following classical Maximin problem that I have been using in my undergraduate teaching for a long long time:

$$z^* := \max_{x \in \mathbb{R}} \min_{y \in \mathbb{R}} \{y^2 + 2xy - x^2\} \tag{59}$$

where $\mathbb{R}$ denotes the real line.

By inspection, the optimal solution is the saddle point $(x^*, y^*) = (0, 0)$ of the expression $y^2 + 2xy - x^2$ over $\mathbb{R}^2$. The graph of this expression is shown in Figure 5.

So what is this idea that the worst-case analysis conducted by Maximin cannot cope with unbounded regions?

Regarding Ben-Haim's (2001, 2006) Info-Gap robustness model,

$$\hat{\alpha}(q, r_c) := \max \{\alpha \geq 0 : r_c \leq R(q, u), \forall u \in \mathcal{U}(\alpha, \tilde{u})\} \tag{60}$$

if indeed α can increase indefinitely, as suggested by Ben-Haim, how come that the robustness model is all about **maximizing** the value of α?

Of course, the answer is that although α can increase indefinitely as far as the regions of uncertainty are concerned, the performance constraint

$$r_c \leq R(q, u) \ , \ \forall u \in \mathcal{U}(\alpha, \tilde{u}) \tag{61}$$

typically imposes an upper bound on α and the task is precisely to determine this upper bound.

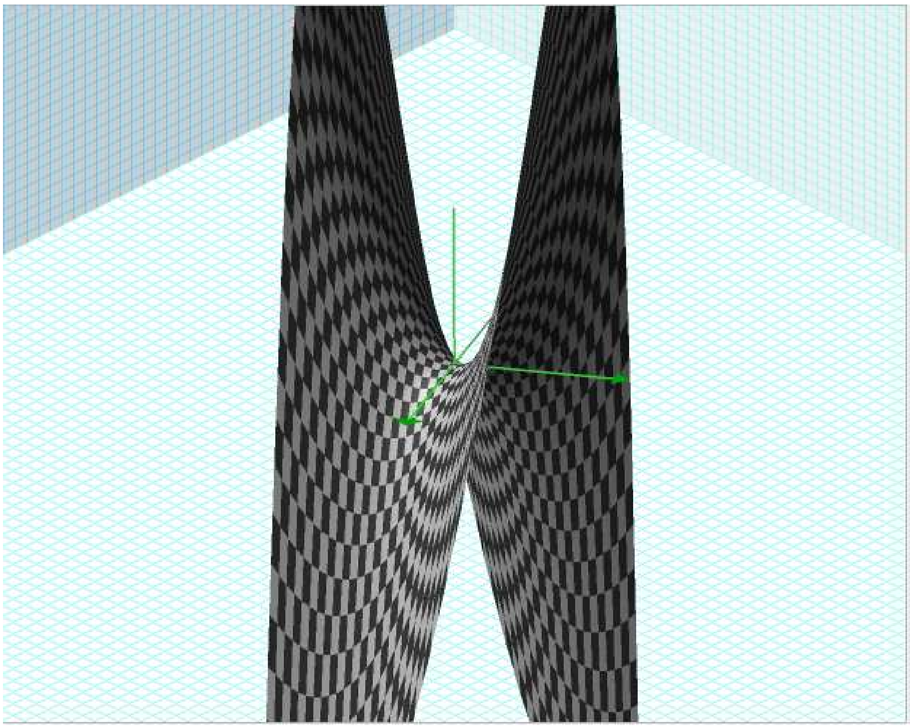


Figure 5: Graph of the expression $y^2 + 2xy - x^2$

The flaw in Ben-Haim's logic can be seen more clearly when examine the classical Maximin format of Info-Gap's robustness model, namely:

$$\hat{\alpha}(q, r_c) := \max_{\alpha \geq 0} \min_{u \in \mathcal{U}(\alpha, \tilde{u})} \alpha \cdot (r_c \preceq R(q, u)) \tag{62}$$

$$= \max_{\alpha \geq 0} \alpha \cdot \min_{u \in \mathcal{U}(\alpha, \tilde{u})} (r_c \preceq R(q, u)) \tag{63}$$

$$= \max_{\alpha \geq 0} G(\alpha) \cdot H(q, \alpha) \tag{64}$$

where

$$G(\alpha) := \alpha \ , \ \alpha \geq 0 \tag{65}$$

$$H(q, \alpha) := \min_{u \in \mathcal{U}(\alpha, \tilde{u})} (r_c \preceq R(q, u)) \ , \ q \in \mathbb{Q}, \alpha \geq 0 \tag{66}$$

observing that the nesting property of the regions of uncertainty implies that for a given q, $H(q, \alpha)$ is a step function of α, as shown in Figure 6(a).

In short, α can increase as much as it wishes: if the robustness of decision q is bounded, then as α increases, sooner or later the objective function of the Maximin model will drop to zero and will remain there forever. Indeed, the robustness of q is precisely the value of α at which $H(q, \alpha)$ attains it maximal value.

It follows then that Ben-Haim apparently has serious misconceptions regarding how Maximin conducts it worst-case analysis and how this is related to the way info-Gap's robustness model operates as a Maximin model.

In fact, the basic misconception is fundamental. For Ben-Haim essentially argues that a function whose argument is unbounded does not have a worst-case (minimum

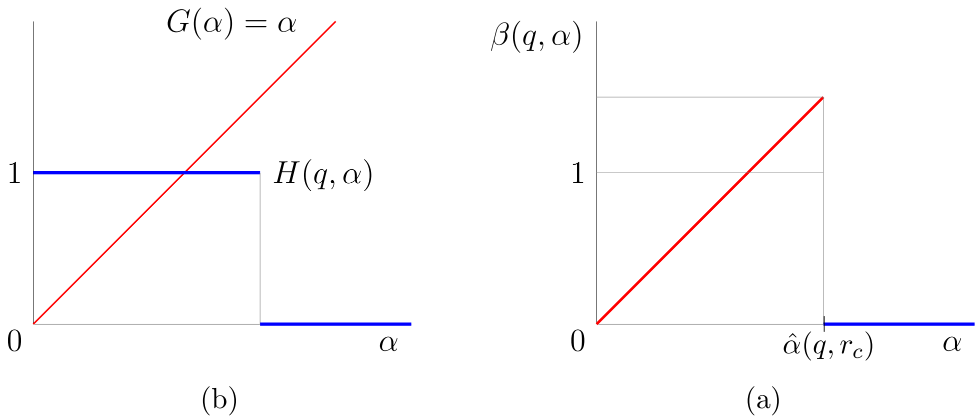


Figure 6: $G(\alpha)$, $H(q, \alpha)$ and $\beta(q, \alpha) = G(\alpha) \cdot H(q, \alpha)$ for a given q.

and/or maximum). So how about, say $\sin(x)$ over the real line? Clearly x is unbounded in this case but $\sin(x)$ is bounded below and above.

Ben-Haim's fundamental error is this: in the framework of Info-Gap's robustness model there is definitely a worst-case. That is, the worst case occurs when the performance constraint $r_c \leq R(q, u)$ is violated. Thus, Ben-Haim's assertion that

> any adverse occurrence is less damaging than some other more extreme event occurring at a larger value of α

is wrong.

If the performance constraint is violated on $\mathcal{U}(\alpha, \tilde{u})$ for some value of α, then a larger value of α will not cause a more extreme event.

Technically speaking, the objective function of the Maximin model representing Info-Gap's robustness model is not monotone increasing/decreasing with α. After α is increased above the value of $\hat{\alpha}(q, r_c)$ by the "max" player (the decision maker), the worst-case is not getting even worst, it stays at the same level. The minimizing player (Nature) cannot do more harm than violating the constraint.

References

[1] Ben-Haim, Y. (1996). *Robust Reliability in the Mechanical Science,* Springer, Berlin.

[2] Ben-Haim, Y. (1999). Design certification with information-gap uncertainty, *Structural Safety,* 2, 269-289.

[3] Ben-Haim, Y. (2001). *Information Gap Decision Theory,* Academic Press, Amsterdam.

[4] Ben-Haim, Y. (2006). *Info-Gap Decision Theory,* Elsevier, Amsterdam.

[5] Ben-Tal A., El Ghaoui, L. and Nemirovski, A. (2006). *Mathematical Programming*, Special issue on *Robust Optimization*, Volume 107(1-2).

[6] Demyanov, V.M. and Malozemov, V.N. (1990). *Introduction to Minimax,* Dover, NY.

[7] Du, D.Z. and Pardalos, P.M. (1995). *Minimax and Applications,* Springer Verlag, Berlin.

[8] Eiselt, H.A., Sandblom, C.L. and Jain, N. (1998). A Spatial Criterion as Decision Aid for Capital Projects: Locating a Sewage Treatment Plant in Halifax, Nova Scotia, *Journal of the Operational Research Society,* 49(1), 23-27.

[9] Eiselt, H.A. and Langley A. (1990). Some extensions of domain criteria in decision making under uncertainty, *Decision Sciences,* 21, 138-153.

[10] French, S.D. (1988). *Decision Theory*, Ellis Horwood, NY.

[11] Galbraith, J.K. (1973) *Designing Complex Organizations,* Addison-Wesley, Reading.

[12] Harsanyi, J.C. (1976). Can the maximin principle serve as a basis for morality?: A critique of John Rawls's theory, Chapter IV (pp. 37-63) in *Essays on Ethics, Social Behavior, and Scientific Explanation,* Springer Verlag, Berlin.

[13] Hurwicz, L. (1951). *A class of criteria for decision-making under ignorance,* Cowles Commission Discussion Paper: Statistics No. 356.

[14] Moffitt, L.J., Stranlund, J.K., and Field, B.C. (2005). Inspections to Avert Terrorism: Robustness Under Severe Uncertainty, *Journal of Homeland Security and Emergency Management,* Volume 2, Issue 3, Article 3, pp. 1-17.

[15] Keynes, J. (1937) The General Theory of Employment, *Quarterly Journal of Economics,* 41(2), 209-223.

[16] Knight, F.H. (1921) *Risk, Uncertainty, and Profit,* Houghton Mifflin Company, Boston, MA.

[17] Kouvelis, P. and Yu, G. (1997). *Robust Discrete Optimization and Its Applications,* Kluwer, Boston.

[18] Lempert, R.J., David G. Groves, D.G., Popper, S.W., and Bankes, S.C. (2006) A General, Analytic Method for Generating Robust Strategies and Narrative Scenarios, *Management Science,* 52(4),514-528.

[19] Loewenstein, G. (1994) The Psychology of Curiosity: A Review and Reinterpretation Psychological Bulletin, 116(1), 75-98.

[20] Los, C.A. (1999) Galton's Error and the Under-Representation of Systematic Risk, *Journal of Banking and Finance,* 23, 1793 - 1829.

[21] Los, C.A. (2003) *Financial Market Risk: Measurement and Analysis,* Routledge/Taylor & Francis Group, London, UK.

[22] Los, C.A. and Tungsong, S. (2008) Investment Model Uncertainty and Fair Pricing", MPRA paper No. 8859, Last modified: 26. May 2008 07:10 (Unpublished)
`http://mpra.ub.uni-muenchen.de/8859/1/MPRA_paper_8859.pdf`

[23] Rawls, J. (2005). *Theory of Justice,* Belknap Press, Cambridge, MA.

[24] Reemstem, R. and Rückmann, J. (1998). *Semi-Infinite Programming,* Kluwer, Boston.

[25] Resnik, M.D. (1987). *Choices: an Introduction to Decision Theory*, University of Minnesota Press, Minneapolis, MN.

[26] Rustem, B. and Howe, M. (2002). *Algorithms for Worst-case Design and Applications to Risk Management*, Princeton University Press, Princeton.

[27] Savage, L.J. (1951). The theory of statistical decision, *Journal of the American Statistical Associatiion,* 46(253), 55-67.

[28] Schneller, G.O., Sphicas, G.P. (1983). Decision making under uncertainty: Starr's domain Criterion, *Theory and Decision,* 15, 321-336.

[29] Sniedovich, M. (2007) "The Art and Science of Modeling Decision-Making Under Severe Uncertainty," Decision-Making in Manufacturing and Services, 1, 1-2, 111-136.

[30] Starr, M.K. (1963). *Product design and decision theory,* Prentice-Hall, Englewood Cliffs, NJ.

[31] Starr, M. K. (1966). A Discussion of Some Normative Criteria for Decision-Making Under Uncertainty, *Industrial Management Review,* 8(1), 71-78.

[32] Tintner, G. (1952). Abraham Wald's contributions to econometrics, *The Annals of Mathematical Statistics,* 23(1), 21-28.

[33] Vladimirou, H. and Zenios, S.A. (1997). Stochastic Programming and Robust Optimization, Chapter 12 in *Advances in Sensitivity Analysis and Parametric Programming,* edited by Gal T, Greenberg HJ, Kluwer, Boston.

[34] von Neumann, J. (1928). Zur theories der gesellschaftsspiele, *Math. Annalen*, Volume 100, 295-320.

[35] von Neumann, J. and Morgenstern, O. (1944). *Theory of Games and Economic Behavior*, Princeton University Press, Princeton.

[36] Wald, A. (1945). Statistical decision functions which minimize the maximum risk, *The Annals of Mathematics,* 46(2), 265-280.

[37] Wald, A. (1950). *Statistical Decision Functions*, John Wiley, NY.

[38] Zhukosky V.I. and Salukvadze M.E. (1994). *The Vector-Valued Maximin*, Academic Press, NY.