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Crystal-Clear Answers to Two FAQs about Info-Gap

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Abstract

This discussion provides crystal-clear answers to the following two central questions regarding the role and place of Info-Gap in decision theory:

Q-1 *Is Info-Gap really a new theory that is so radically different from all classical theories for decision-making under uncertainty? Or is it in fact just a simple instance of Wald's [1945] Maximin model – the most famous model in classical decision theory under severe uncertainty?*

Answer: Info-Gap's generic model is an instance of Wald's Maximin model.

Q-2 *Does Info-Gap generate robust solutions under severe uncertainty? Or does it actually ignore severe uncertainty?*

Answer: Info-Gap does not deal with severe uncertainty – it simply ignores it.

We explain in detail the flaws in Ben-Haim's [2007] answers to these and other related questions.

1 Introduction

Over the past four years I have expressed my constructive criticism on the role and place of Info-Gap theory in decision-making under severe uncertainty. At the end of 2006 I documented my views and made them public. My website

www.ms.unimelb.edu.au/∼ moshe/infogap/infogap.html

provides articles and presentations on this topic.

In this article I address, yet again, the following two central questions regarding the role and place of Info-Gap in decision theory:

FAQ-1 Is Info-Gap really a new theory that is so radically different from all classical theories for decision-making under uncertainty? Or is it in fact just a simple instance of Wald's [1945] Maximin model – the most famous model in classical decision theory under severe uncertainty?

FAQ-2 Does Info-Gap generate robust solutions under severe uncertainty? Or does it actually ignore severe uncertainty?

For anyone unfamiliar with my papers on these topics, these questions are *rhetorical*. As I have shown repeatedly, providing categorical answers to these questions is a very simple task indeed. What is more difficult is to convince Info-Gap aficionados to accept these answers and their implications.

The first question is important because it calls attention to the fact that Info-gap is presented and promoted as a new theory that is radically different from all current theories for Decision making Under Sever Uncertainty. My point is that these claims are made in the absence of a single reference to the most celebrated paradigm for Decision Making Under Sever Uncertainty: Wald's Maximin Model. Indeed, not only is a discussion about Info-Gap's relation to Wald's Maximin Model totally absant from the two editions of the Infogap book. The Maximin is not so much as mentioned in them. This of course is inexcusable.

The situation is in fact more serious. For although there has been significant progress on this front recently to the effect that now Ben-Haim [2007] concedes that Info-Gap is closely related to Maximin, he still insists that Info-Gap's generic model is *not* a Maximin model.

To reiterate, on showing that Info-Gap's generic model is merely an instance of *Wald's classic Maximin model*, not only do we pull the rug out from under the contention that Info-Gap is a new and radically different decision theory, we also call into question Info-Gap's familiarity with the state of the art in decision-making under severe uncertainty and related fields such as *robust optimization* and *worst-case analysis*.

And then there is the related *severe uncertainty/robustness* issue.

Info-Gap is presented and promoted as a methodology that seeks robust decisions under severe uncertainty. I have shown that contrary to claims repeated in the Info-Gap literature, there is no reason to believe that the solutions generated by Info-Gap are robust under severe uncertainty. The fact is that Info-Gap *does not deal with severe uncertainty*, it simply ... *ignores* it.

So this essay is yet another attempt on my part to convince Info-Gap enthusiasts that Info-Gap's generic model is a simple Maximin model and that it does not deal properly with severe uncertainty.

This essay is also motivated by Ben-Haim's [2007] recent compilation of FAQs about Info-Gap that refers to some of the questions that I have raised over the years.

The trouble is, however, that this compilation not only fails to address the SPECIFIC ISSUES that require attention, it exacerbates the situation by providing wrong answers to the two central questions.

The good news, however, is that now I can easily point to and explain the flaws in Ben-Haim's [2007] answers to the two central questions under consideration. In brief:

FAQ-1: Ben-Haim [2007] compares Info-Gap to the "wrong" Maximin model and this leads him to the erroneous conclusion that Info-Gap is not a Maximin model. The short version of the story is as follows.

Generic Info-Gap Model

$$\hat{\alpha}(r_c) := \max_{q\in\mathbb{Q}} \max \left\{ \alpha \geq 0 : r_c \leq \min_{u\in\mathcal{U}(\alpha,\tilde{u})} R(q,u) \right\}$$

"Wrong" Maximin model	"Correct" Maximin model
$r^* := \max_{q\in\mathbb{Q}} \min_{u\in\mathcal{U}(\alpha^\circ,\tilde{u})} R(q,u)$	$z^* := \max_{q\in\mathbb{Q},\alpha\geq 0} \min_{u\in\mathcal{U}(\alpha,\tilde{u})} \alpha\cdot(r_c \preceq R(q,u))$
Ben-Haim [2007]	Sniedovich [2006]

where

$$a \preceq b := \begin{cases} 1 & , \ a \leq b \\ 0 & , \ a > b \end{cases} \ , \ a, b \in \mathbb{R}$$

It is easy to verify that Info-Gap's generic model is equivalent to the "correct" Maximin model.

FAQ-2: Ben-Haim [2007] erroneously argues that because the *domain* of the horizon of uncertainty (α) is *unbounded*, Info-Gap's risk analysis is not local, hence deals with severe uncertainty.

The point to note here is that the largest *admissible* value of this parameter *is implicitly bounded* by the *performance constraint*. Consequently, Info-Gap's risk analysis typically does NOT cover the total region of uncertainty.

In other words, Info-Gap is afflicted by what I call a NO MAN'S LAND SYNDROME. The short version of the story is this:

> Info-Gap's generic model is completely oblivious to what occurs outside the region of uncertainty $\mathcal{U}(\hat{\alpha}(r_c) + \varepsilon, \tilde{u}), \varepsilon > 0$. It conducts a Maximin analysis on $\mathcal{U}(\hat{\alpha}(r_c), \tilde{u})$ but takes no account whatsoever of the remaining part of the total region of uncertainty.

Thus, the conclusion is that Info-Gap conducts a LOCAL risk analysis in the neighborhood of the estimate $\tilde{u}$ which means that it DOES NOT DEAL properly with SEVERE uncertainty. It IGNORES it. For example, consider this concrete instance:

$$\begin{aligned} \tilde{u} &= (0,0) \\ \mathcal{U}(\alpha, \tilde{u}) &= \left\{ u \in \mathbb{R}^2 : |u - \tilde{u}| \leq \alpha \right\} \ , \ \alpha \geq 0 \\ \mathbb{Q} &= \{0, -1\} \\ R(q, u) &= q - u_1 - u_2 \\ r_c &= -\sqrt{50} \end{aligned}$$

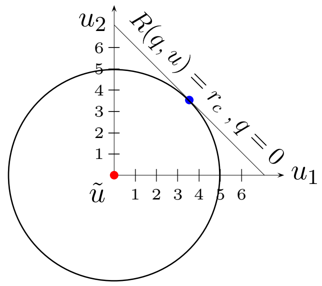


Figure 1: Concrete Instance of an Info-Gap model

The optimal solution generated by this Info-Gap model is $q = 0$ which in turn yields $\hat{\alpha}(r_c) = 5$.

The same results will be generated if we change $R(q, u)$ and let it take ARBITRARY values for $u \notin \mathcal{U}(\hat{\alpha}(r_c) + \varepsilon, \tilde{u}), \varepsilon > 0$ and for $\alpha \geq \hat{\alpha}(r_c) + \varepsilon$ we let $\mathcal{U}(\alpha, \tilde{u})$ be any ARBITRARY set containing $\mathcal{U}(\hat{\alpha}(r_c) + \varepsilon, \tilde{u})$.

That is, the same results are generated regardless of what occurs outside the region of uncertainty $\mathcal{U}(\hat{\alpha}(r_c) + \varepsilon, \tilde{u})$: Info-Gap simply IGNORES what happens there.

Changing the scale of the picture to better reflect the fact that the total region of uncertainty is UNBOUNDED in this example, we obtain the picture shown in Figure 2.

The tiny white circle represents the largest region of uncertainty that actually affects the results generated by Info-Gap, namely $\mathcal{U}(\hat{\alpha}(r_c) + \varepsilon, \tilde{u})$, and the large gray area surrounding it represents INFO-GAP'S NO MAN'S LAND. The results generated by Info-Gap do not depend on how small/large the no man's land is, nor on how the performance function R behaves on this area.

Ben-Haim [2007] claims that this risk analysis is not local in nature!

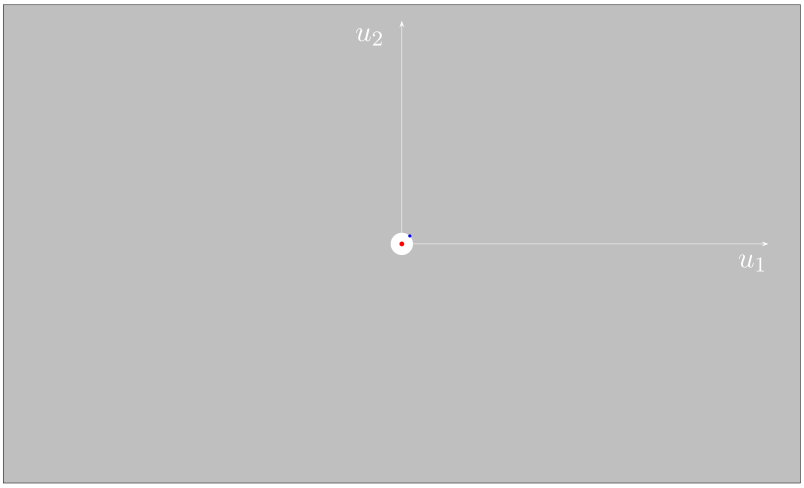


Figure 2: Info-Gap's No Man's Land

In the next two sections I give detailed answers to the two central questions under consideration. Then, in the appendix, I explain in detail the errors in Ben-Haim's [2007] answers to these very questions.

2 FAQ-1: Is Info-Gap a Maximin model?

The answer is a resounding YES!

Here are the two models under examination, namely the well known Wald's Maximin model [1] and the relatively young Info-Gap model:

Wald's Maximin model	Generic Info-Gap model
$z^* := \max_{d\in\mathbb{D}} \min_{s\in S(d)} f(d,s)$	$\hat{\alpha}(r_c) := \max_{q\in\mathbb{Q}} \max \left\{\alpha \geq 0 : r_c \leq \min_{u\in\mathcal{U}(\alpha,\tilde{u})} R(q,u)\right\}$

The question is: what is the relationship between these two mathematical objects?

So consider this:

MAXIMIN THEOREM: *Info-Gap's generic model is an instance of the famous classical Wald's Maximin model.* □

To prove this result we show that a particular choice of the triplet $\mathfrak{M} = (\mathbb{D}, S, f)$ comprising the Maximin model produces an instance of the general model that is *equivalent* to Info-Gap's generic model.

To this end it is instructive to introduce the following *indicator/penalty* function[2] associated with the reward satisficing constraint of the Info-Gap model:

$$\varphi(q,\alpha,u\,|\,r_c) := \begin{cases} \alpha & , \quad r_c \leq R(q,u) \\ -\infty & , \quad r_c > R(q,u) \end{cases} \;,\; q\in\mathbb{Q}, \alpha \geq 0, u\in\mathcal{U}(\alpha,\tilde{u}) \tag{1}$$

This function returns the value α if the reward constraint is satisfied by the triplet (q,α,u) given the stipulated value of r_c. Otherwise it yields the (penalty) value $-\infty$.

[1] Officially $\mathbb{D}$ denotes the *decision space*, $S(d)$ denotes the set of feasible *states* associated with decision d, and f denotes the *objective function* under consideration.

[2] For anyone unfamiliar with this important and useful modeling tool, our old and trusted aid, $-\infty$, is deployed here as a *penalty* to discourage the decision maker from selecting a pair (q,α) that violates the constraint $r_c \leq R(q,u), \forall u\in\mathcal{U}(\alpha,\tilde{u})$. It is deployed routinely in optimization theory for such purposes.

MAXIMIN PROOF.

$$\hat{\alpha}(r_c) := \max_{q\in\mathbb{Q}} \max\left\{\alpha \geq 0 : r_c \leq \min_{u\in\mathcal{U}(\alpha,\tilde{u})} R(q,u)\right\} \tag{2}$$

$$= \max_{q\in\mathbb{Q},\alpha\geq 0} \min_{u\in\mathcal{U}(\alpha,\tilde{u})} \varphi(q,\alpha,u\,|\,r_c) \tag{3}$$

.... QED

In other words, Info-Gap's generic model is equivalent to the instance of Wald's Maximin model that is specified by the following three simple objects:

- *Decision Space:* $\mathbb{D} = \{(q,\alpha) : q \in \mathbb{Q}, \alpha \geq 0\}$.
- *State Space:* $S(d) = \mathcal{U}(\alpha,\tilde{u}),\ d = (q,\alpha) \in \mathbb{D}$.
- *Objective function:* $f(d,s) = \varphi(q,\alpha,u\,|\,r_c),\ d = (q,\alpha) \in \mathbb{D}, s = u \in S(d)$.

Given the simplicity of the proof and the transparent recipe that it provides for translating Info-Gap's generic model into an equivalent Maximin model, it is puzzling that Info-Gap is still considered to be different from Maximin.

Indeed, it is amusing to read the discussion in the Info-Gap literature as to whether Info-Gap is similar to but different from, or different from but similar to, Maximin.

In the appendix I explain the fundamental flaw in Ben-Haim's [2007] analysis of the relationship between Info-Gap and Maximin that mislead him to conclude that Info-Gap is not an instance of Wald's Maximin model.

3 FAQ-2: Does Info-Gap deal properly with severe uncertainty?

The answer is a resounding NO!

To see more clearly why this is so let us examine a peculiar feature of Info-Gap's generic model and its ramifications. This feature has to do with the fact that Info-Gap's generic model is *completely oblivious* to what "happens" outside the largest admissible region of uncertainty. That is, it takes no notice of the area outside the region of uncertainty associated with the maximum robustness, namely the area outside the region $\mathcal{U}(\hat{\alpha}(r_c),\tilde{u})$. We shall refer to this feature of Info-Gap as the *No Man's Land Syndrome*.

This is illustrated graphically in Figure 3. The rectangle represents the total region of uncertainty and the light-gray inner circle represent the region of uncertainty associated with the maximum robustness $\hat{\alpha}(r_c)$, namely the region $\mathcal{U}(\hat{\alpha}(r_c),\tilde{u})$. Hence, the dark gray area surrounding the light circle represents the region of uncertainty ignored by Info-Gap's analysis. We shall refer to this area of the total region of uncertainty as the *no man's land*. Note that ε can be arbitrarily small, but it must be strictly positive.

The results generated by Info-Gap's generic model are not affected by how small/large its no man's land is, nor by how the performance function R behaves in this area.

Although this fact stares one in the face, we nevetheless prove it formally. So let $\mathfrak{U}$ denote the *total region of uncertainty* in question and consider this:

DEFINITION: We refer to

$$\overline{\mathcal{U}}(\hat{\alpha}(r_c),\tilde{u}) := \mathfrak{U} \setminus \mathcal{U}(\hat{\alpha}(r_c)+\varepsilon,\tilde{u})\ ,\ \varepsilon > 0 \tag{4}$$

as Info-Gap's NO MAN'S LAND.

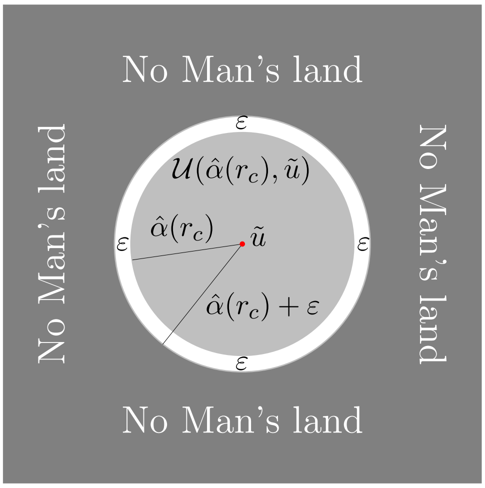


Figure 3: Illustration of Info-Gap's NO MAN'S LAND SYNDROME

By definition then, $\overline{\mathcal{U}}(\hat{\alpha}(r_c),\tilde{u})$ denotes the area of the total region of uncertainty $\mathfrak{U}$ that is not covered by the region $\mathcal{U}(\hat{\alpha}(r_c)+\varepsilon,\tilde{u})$. The ε is required to ensure that $\mathcal{U}(\hat{\alpha}(r_c)+\varepsilon,\tilde{u})$ contains a u' such that $R(q,u')$ violates the performance constraint. It can be arbitrarily small, but must be strictly positive[3].

NO MAN'S LAND THEOREM: *Info-Gap's generic model is* INVARIANT *with its no man's land. That is, Info-Gap's generic model generates the same results regardless of how small/large the no man's land* $\overline{\mathcal{U}}(\hat{\alpha}(r_c),\tilde{u})$ *is, and how* R *behaves on* $\overline{\mathcal{U}}(\hat{\alpha}(r_c),\tilde{u})$.

NO MAN'S LAND PROOF: *This is an immediate consequence of the* NESTING *property of the regions of uncertainty, namely of the fact that*

$$\alpha > \alpha' \longrightarrow \mathcal{U}(\alpha',\tilde{u}) \subseteq \mathcal{U}(\alpha,\tilde{u}) \tag{5}$$

observing that this property entails that

$$r_c > \min_{u\in\mathcal{U}(\alpha,\tilde{u})} R(q,u) \ , \forall q \in \mathbb{Q}, \alpha > \hat{\alpha}(r_c) \tag{6}$$

.... QED

The following is an immediate implication of this result.

INVARIANCE THEOREM: *The results generated by Info-Gap's generic model are* INVARIANT *with the* SIZE *of the total region of uncertainty* $\mathfrak{U}$. *That is, the model generates the same results if we change* $\mathfrak{U}$ *to any* $\mathfrak{U}'$ *such that* $\mathcal{U}(\hat{\alpha}(r_c)+\varepsilon,\tilde{u}) \subseteq \mathfrak{U}', \varepsilon > 0$.

This feature is illustrated graphically in Figure 4 where the small circle represents $\mathcal{U}(\hat{\alpha}(r_c),\tilde{u})$. Info-Gap's analysis and results are the SAME for all cases where the total region of uncertainty ($\mathfrak{U}$) contains $\mathcal{U}(\hat{\alpha}(r_c)+\varepsilon,\tilde{u})$. The SAME analysis is conducted for $\mathfrak{U}$, $\mathfrak{U}'$ and $\mathfrak{U}''$. In fact, we can increase the size of the region of uncertainty *indefinitely* and this will have no impact whatsoever on the results generated by Info-Gap.

So much then for a methodology claiming to seek robust decisions under SEVERE uncertainty!

[3]If you are allergic to epsilons and suchlike, simply ignore the technicalities and regard it as a device used to ensure that the results generated by Info-Gap's generic model are not influenced by how R behaves on $\overline{\mathcal{U}}(\hat{\alpha}(r_c),\tilde{u})$.

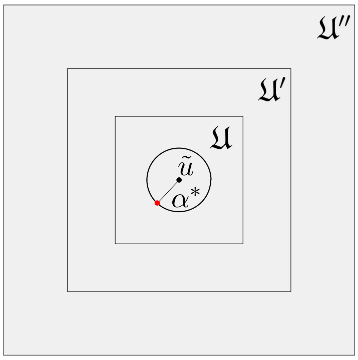


Figure 4: Illustration of the INVARIANCE THEOREM, $\alpha^* = \hat{\alpha}(r_c)$.

The following simple example illustrates this point. Let

$$\mathfrak{U} = \mathbb{R}^2 \tag{7}$$

$$\tilde{u} = (0,0) \tag{8}$$

$$\mathcal{U}(\alpha,\tilde{u}) = \{u \in \mathfrak{U} : |u - \tilde{u}| \leq \alpha\} \ , \ \alpha \geq 0 \tag{9}$$

$$= \left\{u \in \mathbb{R}^2 : u_1^2 + u_2^2 \leq \alpha^2\right\} \tag{10}$$

$$\mathbb{Q} = \{1,2\} \tag{11}$$

$$R(q,u) = \begin{cases} 6 - q\left(u_1^2 + u_2^2\right) & , \quad u_1^2 + u_2^2 \leq 3^2 \\ \text{negotiable} & , \quad u_1^2 + u_2^2 > 3^2 \end{cases} \tag{12}$$

$$r_c = 2 \tag{13}$$

Since clearly (by inspection) the maximum robustness is not greater than 3, we are indifferent to the result of the negotiations that will eventually determine the values that R will take for u outside $\mathcal{U}(3,\tilde{u})$. Hence,

$$\hat{\alpha}(r_c) := \max_{q \in \{1,2\}} \max \left\{ \alpha \geq 0 : 2 \leq \min_{u_1^2 + u_2^2 \leq \alpha^2} \left\{ 6 - q\left(u_1^2 + u_2^2\right)\right\} \right\} \tag{14}$$

Then by inspection, the optimal decision is $q^* = 1$ and the resulting (optimal) robustness is equal to $\hat{\alpha}(r_c) = 2$. Therefore, in this case Info-Gap's no man's land is

$$\overline{\mathcal{U}}(\hat{\alpha}(r_c),\tilde{u}) = \mathfrak{U} \setminus \mathcal{U}(\hat{\alpha}(r_c) + \varepsilon, \tilde{u}) \tag{15}$$

$$= \mathbb{R}^2 \setminus \mathcal{U}(2 + \varepsilon, (0,0)) \tag{16}$$

$$= \mathbb{R}^2 \setminus \left\{u \in \mathbb{R}^2 : u_1^2 + u_2^2 \leq (2+\varepsilon)^2\right\} \tag{17}$$

To keep things simple, let $\varepsilon = 1$, in which case we have

$$\overline{\mathcal{U}}(\hat{\alpha}(r_c),\tilde{u}) = \mathbb{R}^2 \setminus \left\{u \in \mathbb{R}^2 : u_1^2 + u_2^2 \leq 3^2\right\} \tag{18}$$

$$= \left\{u \in \mathbb{R}^2 : u_1^2 + u_2^2 > 3^2\right\} \tag{19}$$

We dedicate the next page of this article to a graphic display of the situation. The gray disc represents the safety area associated with $\varepsilon = 1$.

In the appendix I explain in detail the flaws in Ben-Haim's [2007] analysis that mislead him to conclude that Info-Gap's robustness is not local in nature.

4 An illustrative example

I have observed for some time now that Info-Gap users are greatly surprised when shown how absurd the results generated by Info-Gap can be. So, the following simple illustrative example is designed to drive this home.

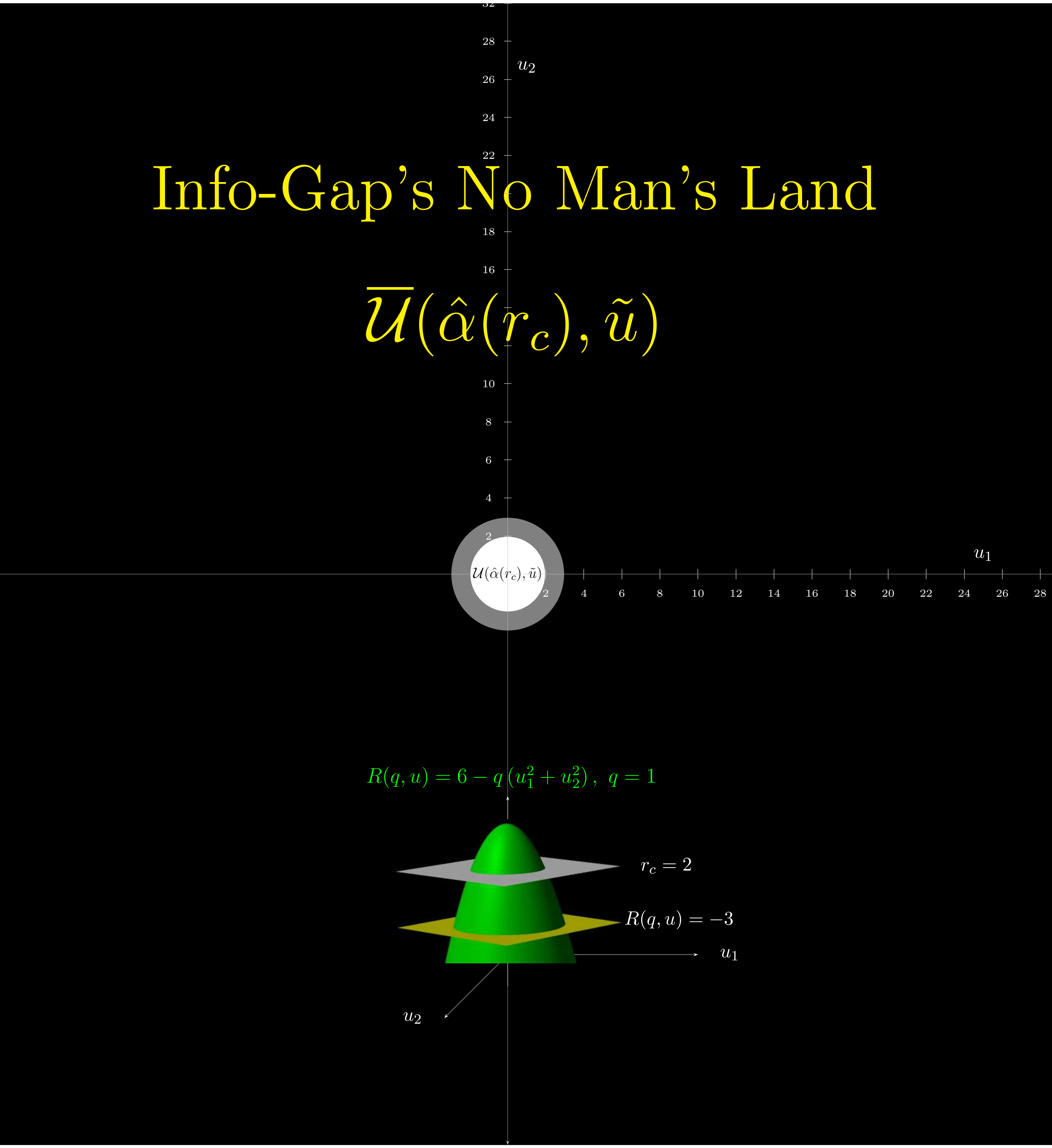


Figure 5: The Situation Room for $\overline{\mathcal{U}}(\hat{\alpha}(r_c),\tilde{u})$.

It reads as follows:

$$\mathfrak{U} = [-7,7] \tag{20}$$
$$\tilde{u} = 0 \tag{21}$$
$$\mathcal{U}(\alpha,\tilde{u}) = \{u \in [-7,7] : |u-\tilde{u}| \leq \alpha\} \ , \ \alpha \geq 0 \tag{22}$$
$$= \begin{cases} [-\alpha,\alpha] & , \ 0 \leq \alpha \leq 7 \\ [-7,7] & , \ \alpha > 7 \end{cases} \tag{23}$$
$$\mathbb{Q} = \{1,2\} \tag{24}$$
$$R(q,u) = \begin{cases} 6-|u| & , \ q=1 \\ 5-0.75|u| & , \ q=2 \end{cases} \tag{25}$$
$$r_c = 2 \tag{26}$$

Formally we thus have:

$$\hat{\alpha}(r_c) := \max_{q\in\{1,2\}} \max \left\{ \alpha \geq 0 : 2 \leq \min_{u\in\mathcal{U}(\alpha,0)} R(q,u) \right\} \tag{27}$$

Since it is clear that $u > 4$ violates the performance constraint for both $q = 1$ and $q = 2$, it follows that we can safely restrict the analysis to $\alpha \leq 4$, hence

$$\hat{\alpha}(r_c) = \max_{q\in\{1,2\}} \max \left\{ 4 \geq \alpha \geq 0 : 2 \leq \min_{-\alpha\leq u\leq\alpha} R(q,u) \right\} \tag{28}$$

Both decisions are optimal, yielding $\hat{\alpha}(r_c) = 4$. There are two critical (worst case) values for u in $\mathcal{U}(\hat{\alpha}(r_c),\tilde{u})$, namely -4 and 4, and the no man's land region here is the set $\overline{\mathcal{U}}(\hat{\alpha}(r_c),\tilde{u}) = \{u \in \mathbb{R} : 4+\varepsilon < |u| \leq 7\}$ for some $\varepsilon > 0$. In other words, Info-Gap "could not care less" about what goes on in the region $\{u \in \mathbb{R} : 4+\varepsilon < |u| \leq 7\}$. This model is described graphically in Figure 6 for some small ε, say $\varepsilon = 0.00001$ (just kidding!).

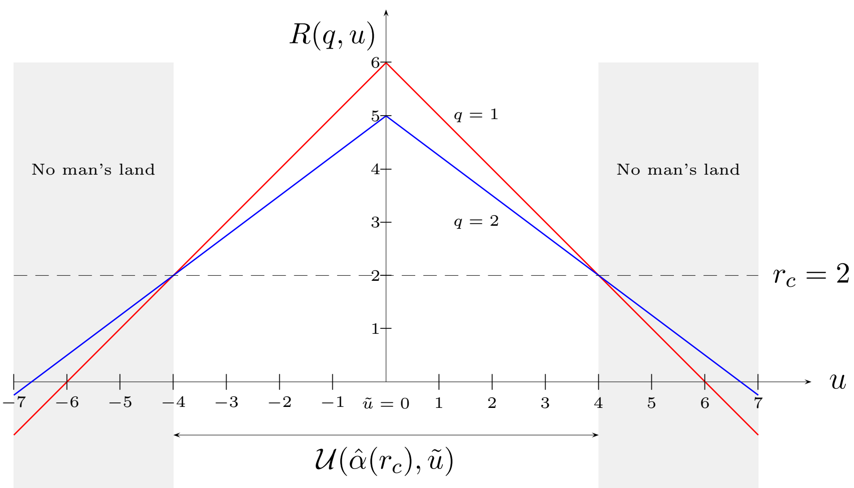


Figure 6: Illustrative Example

As is amply clear, in this case Info-Gap will generate the same results if we change the given total region of uncertainty from $\mathfrak{U} = [-7,7]$ to say $\mathfrak{U} = \mathbb{R}$ and let the performance function R take ARBITRARY values on the new no man's land region $\{u \in \mathbb{R} : |u| \geq 4+\varepsilon\}$.

In short, in this particular example, Info-Gap is *totally blind* to what occurs outside the region of uncertainty $[-4,4]$.

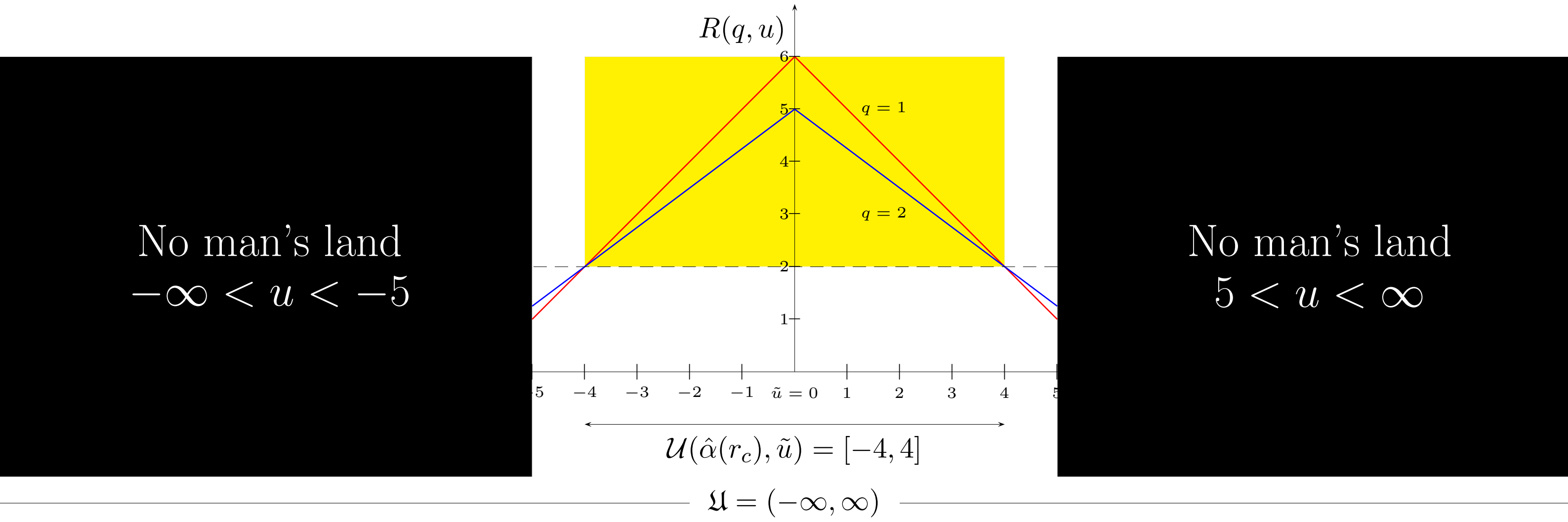


Figure 7: An intriguing Info-Gap model and the results it generates.

Next consider the intriguing Info-Gap model specified by:

$$\mathfrak{U} = \mathbb{R} \tag{29}$$
$$\tilde{u} = 0 \tag{30}$$
$$\mathcal{U}(\alpha, \tilde{u}) = \{u \in \mathbb{R} : |u - \tilde{u}| \leq \alpha\} \ , \ \alpha \geq 0 \tag{31}$$
$$= [-\alpha, \alpha] \tag{32}$$
$$\mathbb{Q} = \{1, 2\} \tag{33}$$
$$r_c = 2 \tag{34}$$
$$R(q,u) = \begin{cases} 6 - |u| & , \ q = 1, -5 \leq u \leq 5 \\ 5 - 0.75|u| & , \ q = 2, -5 \leq u \leq 5 \\ \textit{No Man's Land} & , \ |u| > 5 \end{cases} \tag{35}$$

Figure 7 shows the basic ingredients of the model and the results it generates. Note that as before, both decisions are optimal, $\hat{\alpha}(r_c) = 4$, and formally $\varepsilon = 1$.

But to give the reader a better perspective on what goes on here, it is instructive to present the results graphically using a more realistic scaling. This is difficult because the No Man's Land is unbounded and the region covered by the Info-Gap analysis, namely $\mathcal{U}(\hat{\alpha}(r_c), \tilde{u}) = [-4, 4]$, is bounded. A compromise is shown in Figure 8.

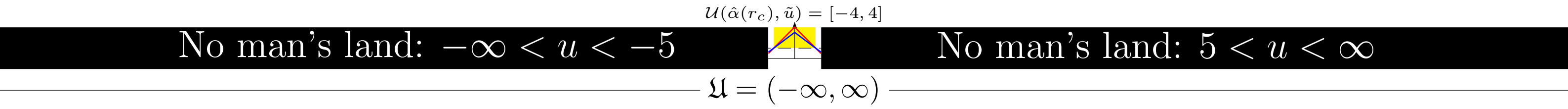


Figure 8: An intriguing Info-Gap model and the results it generates.

This analysis demonstrates that we are dealing here with either pure magic or pure nonsense, or both: we are dealing here with a model whose *raison d'être* is to tackle SEVERE uncertainty, but the model altogether IGNORES the behavior of the performance function R on a huge (unbounded) no man's land ($|u| > 5$), comprising practically the entire region of uncertainty under consideration.

Yet, Info-Gap contends that it properly handles SEVERE uncertainty. What is more, we are expected to accept that the Info-Gap solution is ROBUST, even though under severe uncertainty the estimate $\tilde{u}$ is a POOR indication of the true value of u and is likely to be SUBSTANTIALLY WRONG.

Isn't this voodoo[4] decision-making?

Remark:
Some readers may object that I am using a contrived, unrealistic example especially concocted to support my criticism of Info-Gap. In particular, my choice of a problem where the total region of uncertainty $\mathfrak{U}$ is UNBOUNDED may be dismissed as mischievous.

So let us remind ourselves that according to Ben-Haim [2007, p. 2]:

> 2. An info-gap analysis is not based on an estimate of the true horizon of uncertainty. That is, the info-gap model of uncertainty is **not** a single set, $\mathcal{U}(\alpha, \tilde{u})$. Rather, an info-gap model is a family of nested sets, $\mathcal{U}(\alpha, \tilde{u})$ for all $\alpha \geq 0$. The family of sets is usually unbounded[1]. Thus an info-gap model is not a "local analysis of risk" since the family of sets expands, usually boundlessly, as the unknown horizon of uncertainty, α, grows. Info-gap theory is **not** a worst-case analysis, since there is no known worst case in an info-gap model of uncertainty.
>
> [1] The family of sets is bounded only when there is a physical or definitional limit of the range of variation, such as probabilities not being larger than unity, or masses not being negative.

Clearly then, an UNBOUNDED total region of uncertainty should be regarded as the "usual" case as far as Info-Gap is concerned. I therefore do not see any difficulty in using unbounded regions of uncertainty in my illustrative examples. To the contrary, given Ben-Haim's [2007] unequivocal statement about the scope of Info-Gap's region of uncertainty, I find it necessary to consider examples featuring unbounded regions of uncertainty.

5 Conclusions

The bottom line then is this:

- Info-Gap's generic model is neither new nor radically different. It is a simple instance of the famous Wald's Maximin model.
- Info-Gap's uncertainty model does not deal properly with severe uncertainty, it simply ignores it.
- There is no ground to contend that the solutions generated by Info-Gap are robust under severe uncertainty.
- The sooner the Info-Gap community accepts these facts, the better.
- This is long overdue.
- Info-Gap users should consult the *worst-case analysis* and *robust optimization* literature (eg Rosenblatt and Lee [1987], Kouvelis and Yu [1997], Rustem and Howe [2002], Ben-Tal et al [2006]) for guidance on robust optimization under severe uncertainty.

[4] Here voodoo means "a belief, theory or method that lacks sufficient evidence or proof." (Encarta World English Dictionary)

My experience with Info-Gap over the past four years has confirmed my long held views on the role of MATHEMATICAL MODELING in decision-making, particularly under severe uncertainty.

Indeed, Info-Gap is an excellent example of how wrong things can go due to a failure to appreciate the **mathematical modeling** aspects of MAXIMIN, WORST-CASE ANALYSIS AND SEVERE UNCERTAINTY.

Appendix

In this appendix I explain in detail some of the flaws in Ben-Haim's [2007] answers to the two central questions under consideration.

A Is Info-Gap's generic model a Maximin model?

Yes, it definitely is!

Ben-Haim [2007, p. 4] now concedes that Info-Gap's generic model is closely related to Maximin:

> Info-gap robust-satisficing and the max-min decision strategy are not the same, but there is a close relation between them.

The point to note here is that to substantiate this claim Ben-Haim does not refute my argument that Info-Gap is a SPECIFIC INSTANCE of Wald's Maximin model. Instead he compares Info-Gap to a different ("wrong") Maximin model that he constructed for this purpose.

Although Ben-Haim [2007] does not explicitly define the Maximin model he has in mind, a bit of detective work reveals that this model is of the following form:

$$r^* := \max_{q\in\mathbb{Q}} \min_{u\in\mathcal{U}(\alpha^\circ,\tilde{u})} R(q,u) \tag{36}$$

where α° denotes the "estimated uncertainty" (level of robustness) employed to control the robustness required of (or stipulated by) the Maximim model.

It is patently clear that this model seeks to maximize the worst (smallest) value of the *reward* $R(q,u)$.

Now, this leaves one gasping!

Since Info-Gap's model seeks to maximize robustness, *common sense* dictates that it should be compared to a Maximin model that also seeks to maximize robustness, indeed the same type of robustness. What is the point of comparing Info-Gap's generic model – whose aim is to maximize ROBUSTNESS – to a Maximin model whose aim is to maximize REWARD?!

> The flaw in Ben-Haim's [2007] analysis is fundamental: it compares *kangaroos* with *Vegemite*. That is, the analysis compares a model designed to maximize *robustness* with a model designed to maximize *reward* only to discover that they are different!

Employing Ben-Haim's [2007] logic, we can effortlessly prove all kind of interesting propositions. For example, the proof that the expression $x+x^2, x \in \mathbb{R}$ is **not** a polynomial would be straightforward. It would run as follows: As we all know, $P(x) := x$ is a polynomial. Since clearly $x + x^2$ is not equivalent to $P(x)$, we must conclude that $x + x^2$ is not a polynomial.

Another odd feature of Ben-Haim's [2007] Maximin model is that it takes no notice of the crucial role that the performance constraint $r_c \leq R(q,u), \forall u \in \mathcal{U}(\alpha,\tilde{u})$ plays in the Info-Gap model.

So one cannot help but wonder about the logic behind Ben-Haim's [2007] comparison of two models that do not address the same goal nor do they incorporate the same requirements. If we do not care about these details, we can easily construct 20465765 (and more) maximin models that are different from the Info-Gap model.

This, however, is not the issue here.

The issue here is whether a Maximin model exists that *does precisely what the Info-Gap model does*. Therefore, the only valid comparison in this context is one where the Maximin model seeks to *maximize robustness*, rather than reward.

> The MAXIMIN PROOF shows in no uncertain terms that when Info-Gap is compared with a proper Maximin model, this model does exactly what the Info-Gap model does, hence Info-Gap is an instance of *Wald's Maximin model*.

In summary then:
The question regarding the relationship between Info-Gap and Maximin is *not* whether there exist Maximin models that are *different* from the Info-Gap model. Of course there are such models. After all, there are numerous instances of Wald's generic Maximin model and Ben-Haim's [2007] model is just one such case.

Rather, the question is whether there exists a Maximin model that is *equivalent* to the generic Info-Gap model. And the answer to this question is a resounding YES.

In fact, for our purposes it is instructive to rewrite the equivalent Maximin model (3) as follows:

$$z^* := \max_{q\in\mathbb{Q},\alpha\geq 0} \min_{u\in\mathcal{U}(\alpha,\tilde{u})} \alpha\cdot(r_c \preceq R(q,u)) \tag{37}$$

where

$$a \preceq b := \begin{cases} 1 & , \quad a \leq b \\ 0 & , \quad a > b \end{cases} \ , \ a,b \in \mathbb{R} := (-\infty,\infty) \tag{38}$$

assuming (just a technicality) that $r_c \leq R(q,\tilde{u}), \forall q \in \mathbb{Q}$.

This formulation of the objective function of the equivalent Maximin model is more explicit about what the Maximin model does and it underscores its obvious equivalence to the generic Info-Gap model. This then is the picture:

Generic Info-Gap Model

$$\hat{\alpha}(r_c) := \max_{q\in\mathbb{Q}} \max\left\{\alpha \geq 0 : r_c \leq \min_{u\in\mathcal{U}(\alpha,\tilde{u})} R(q,u)\right\}$$

Wrong Maximin model	Correct Maximin model
$r^* := \max_{q\in\mathbb{Q}} \min_{u\in\mathcal{U}(\alpha^{\circ},\tilde{u})} R(q,u)$	$z^* := \max_{q\in\mathbb{Q},\alpha\geq 0} \min_{u\in\mathcal{U}(\alpha,\tilde{u})} \alpha\cdot(r_c \preceq R(q,u))$
Ben-Haim [2007]	Sniedovich [2006]

And if you are allergic to *multiplicative* objective functions, you may prefer the following equivalent "additive" Maximin model:

$$z^* := \max_{q\in\mathbb{Q},\alpha\geq 0} \min_{u\in\mathcal{U}(\alpha,\tilde{u})} \{\alpha - \psi(q,u,\alpha)\} \tag{39}$$

where

$$\psi(q,u,\alpha) := \begin{cases} 0 & , \quad r_c \leq R(q,u) \\ \alpha & , \quad r_c > R(q,u) \end{cases} \ , \ q \in \mathbb{Q}, u \in \mathcal{U}(\alpha,\tilde{u}), \alpha \geq 0 \tag{40}$$

To sum up, Info-Gap's decision model is neither new nor radically different from existing models. It is a simple instance of the famous *Wald's Maximin model*.

I should also point out that Ben-Haim's [2007a, slide 47] even goes so far as to claim that Info-Gap's robust-satisficing approach "beats max-min in competition".

What can one say to that?!

B Does Info-Gap deal properly with severe uncertainty?

No, it does not!

In this section I address some of the flaws in Ben-Haim's [2007] answer to the "Severe uncertainty/Robustness" question under consideration.

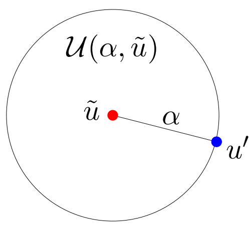
B.1 Is Info-Gap's robustness analysis local?

Yes, it is!

How else would you describe an analysis that DEFINES ROBUSTNESS as the maximum admissible DEVIATION FROM A GIVEN FIXED POINT, namely from $\tilde{u}$?!?!

Info-Gap's robustness analysis is INHERENTLY LOCAL precisely because it defines the robustness of a decision as the maximum admissible DEVIATION from a GIVEN FIXED POINT $\tilde{u}$ that the decision can withstand. This means that Info-Gap's robustness analysis is local even if $\hat{\alpha}(r_c)$ is relatively large: it is conducted in the LOCALE of a given fixed point.

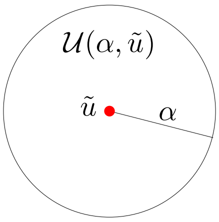
The local nature of Info-Gap's robustness is manifested in the phrasing of the question that Info-Gap is supposed to answer:



How much can u' deviate from $\tilde{u}$ – measured by α – without causing a violation in the performance constraint $r_c \leq R(q,u), \forall u \in \mathcal{U}(\alpha,\tilde{u})$?

Figure 9: On the local nature of Info-Gap's robustness analysis

In fact, it is more instructive to phrase the question without any reference to the point u'. That is, the local nature of Info-Gap analysis is clearly exposed by observing that $\hat{\alpha}(r_c)$ represents the maximum admissible deviation from $\tilde{u}$.



How much can we deviate from $\tilde{u}$ – as measured by α – without causing a violation in the performance constraint $r_c \leq R(q,u), \forall u \in \mathcal{U}(\alpha,\tilde{u})$?

Figure 10: On the local nature of Info-Gap's robustness analysis

Thus, consider the following phrasing of the question that Info-Gap's analysis is supposed to answer:

The related claim that Info-Gap's analysis is not "necessarily" local confuses two facts, namely:

· Info-Gap's robustness analysis is LOCAL in nature.

· In certain situations $\hat{\alpha}(r_c)$ can be relatively LARGE.

These represent two different aspects of the Info-Gap model that should be handled with care.

(i) The local nature of Info-Gap's robustness analysis.
This is a consequence of the NESTED structure of the regions of uncertainty. To see that this is so, let $\underline{\alpha}(u,\tilde{u})$ denote the distance of point $u \in \mathfrak{U}$ from the estimate $\tilde{u}$, namely set

$$\underline{\alpha}(u,\tilde{u}) := \min\{\alpha \geq 0 : u \in \mathcal{U}(\alpha,\tilde{u})\}\ ,\ \ u \in \mathfrak{U} \tag{41}$$

That is, $\underline{\alpha}(u,\tilde{u})$ denotes the smallest value of α such that $u \in \mathcal{U}(\alpha,\tilde{u})$. We can regard $\underline{\alpha}(u,\tilde{u})$ as the "distance of u from $\tilde{u}$".

The distinction between points in $\mathcal{U}(\hat{\alpha}(r_c),\tilde{u})$ and points in the associated NO MAN'S LAND $\overline{\mathcal{U}}(\hat{\alpha}(r_c),\tilde{u})$ is their distance from the estimate $\tilde{u}$:

$$u \in \overline{\mathcal{U}}(\hat{\alpha}(r_c),\tilde{u}) \longleftrightarrow \underline{\alpha}(u,\tilde{u}) > \hat{\alpha}(r_c) \tag{42}$$

$$u \in \mathcal{U}(\hat{\alpha}(r_c),\tilde{u}) \longleftrightarrow \underline{\alpha}(u,\tilde{u}) \leq \hat{\alpha}(r_c) \tag{43}$$

In short, Info-Gap's robustness analysis "IGNORES" points in the region of uncertainty whose distance from $\tilde{u}$ is larger than $\hat{\alpha}(r_c)$. As shown in Figure 3, Figure 4 and Figure 9, this is a "local" analysis par excellence.

(ii) The "size" of the optimal region of uncertainty $\mathcal{U}(\hat{\alpha}(r_c),\tilde{u})$.
Needless to say, the actual value of $\hat{\alpha}(r_c)$ is problem dependent and can vary significantly from problem to problem. But as explained above, this does not alter the fact that Info-Gap's robustness analysis is INHERENTLY LOCAL.

Interestingly, according to Ben-Haim' [2007, p. 2]:

> 2. An info-gap analysis is not based on an estimate of the true horizon of uncertainty. That is, the info-gap model of uncertainty is **not** a single set, $\mathcal{U}(\alpha,\tilde{u})$. Rather, an info-gap model is a family of nested sets, $\mathcal{U}(\alpha,\tilde{u})$ for all $\alpha \geq 0$. The family of sets is usually unbounded[1]. Thus an info-gap model is not a "local analysis of risk" since the family of sets expands, usually boundlessly, as the unknown horizon of uncertainty, α, grows. Info-gap

> theory is **not** a worst-case analysis, since there is no known worst case in an info-gap model of uncertainty.
>
> ---
> [1] The family of sets is bounded only when there is a physical or definitional limit of the range of variation, such as probabilities not being larger than unity, or masses not being negative.
>
> Ben-Haim [2007, p. 2]

This is odd. Very odd.

To see this, let us look again at Info-Gap's generic model:

$$\hat{\alpha}(r_c) := \max_{q\in\mathbb{Q}} \ \max\left\{\alpha \geq 0 : r_c \leq \min_{u\in\mathcal{U}(\alpha,\tilde{u})} R(q,u)\right\} \tag{44}$$

If indeed α is unbounded (from above) and can grow indefinitely, then how is it that the generic Info-Gap model sets the *maximization* of α to be its goal?!?

The flaw in Ben-Haim's [2007] argument is this.

The statement $\alpha \geq 0$ in no way entails that all the regions of uncertainty $\mathcal{U}(\alpha,\tilde{u}), \alpha \geq 0$ are actually examined by Info-Gap and that they affect the results generated by the model (44). The point is – as clearly stated by Info-Gap's generic model – that the admissible values of α must also satisfy the performance requirement

$$r_c \leq R(q,u) \ , \ \forall u \in \mathcal{U}(\alpha,\tilde{u}) \tag{45}$$

The largest admissible value of α is $\hat{\alpha}(r_c)$, hence the largest region of uncertainty considered by the Info-Gap model is $\mathcal{U}(\hat{\alpha}(r_c),\tilde{u})$. In other words, the analysis conducted by the Info-Gap model is CONFINED to the immediate neighborhood of $\tilde{u}$ specified by $\mathcal{U}(\hat{\alpha}(r_c),\tilde{u})$ as shown in Figure 11, where the regions of uncertainty are represented by circles of various sizes, ALL CENTERED AT THE SAME POINT, namely at $\tilde{u}$. The largest circle represents $\alpha = \hat{\alpha}(r_c)$.

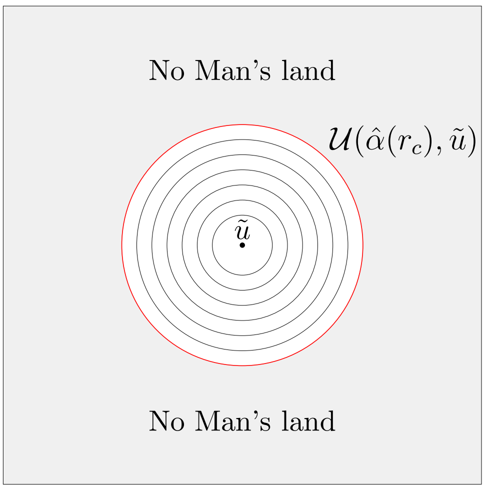


Figure 11: The local nature of Info-Gap's robustness analysis

Note that the neighborhood shown in this picture is finite in size, yet it consists of infinitely many regions of uncertainty, whose sizes vary (continuously) from 0 to $\hat{\alpha}(r_c)$.

In summary, the fact that the *formulation* of the Info-Gap model involves infinitely many region of uncertainty in no way implies that the size of the neighborhood of $\tilde{u}$ that is actually involved in the analysis of a given problem is unbounded. In fact, in all cases where the results are not trivial, the size of the largest neighborhood examined by the model is finite and is equal to the maximum robustness $\hat{\alpha}(r_c)$.

Info-Gap's robustness analysis is therefore INHERENTLY LOCAL, centered as it is at a SINGLE POINT $\tilde{u} \in \mathfrak{U}$. Indeed, usually the results generated by Info-Gap's generic model vary with $\tilde{u}$. As I pointed out on many occasions, it is regrettable that this fact is not manifested in the notation used by Info-Gap. It would have been far more informative to incorporate $\tilde{u}$ in $\hat{\alpha}(q, r_c)$, say by denoting the robustness of decision q by $\hat{\alpha}(q|r_c, \tilde{u})$.

Remarks:

(i) As indicated above, Ben-Haim [2007] goes out of his way to point out that Info-Gap's total uncertainty region, namely $\mathfrak{U}$, is "...usually unbounded ...".

This means that "usually" the largest region of uncertainty participating in the Info-Gap analysis, namely $\mathcal{U}(\hat{\alpha}(r_c), \tilde{u})$, is extremely small in size compared to the size of $\mathfrak{U}$. In fact in this "usual" case Info-Gap's no man's land, namely $\overline{\mathcal{U}}(\hat{\alpha}(r_c), \tilde{u})$, is unbounded and is much larger in size than $\mathcal{U}(\hat{\alpha}(r_c), \tilde{u})$ (see Figure 5).

(ii) Figure 12 illustrates the distinction between local and non-local "coverage". The total area covered in both cases (local and non-local) is the same. However, in the local case the region covered is concentrated in the immediate neighborhood of the estimate $\tilde{u}$, whereas in the non-local case the area covered is split into 12 smaller sub-regions that are spread all over the total region of uncertainty.

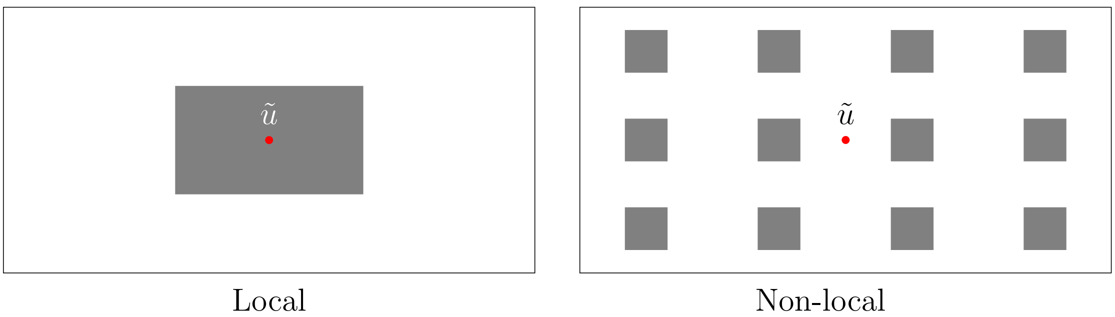


Figure 12: Local vs Non-local coverage

(iii) A local analysis of the type conducted by Info-Gap makes sense if there is reason to believe that the true value of u is much more likely to be close to the value of the estimate $\tilde{u}$ than to other values of u in the total region of uncertainty. But Info-Gap assumes the precise opposite: the estimate $\tilde{u}$ is a poor indicator of the true value of u and is likely to be substantially wrong. In other words, there is a sharp contradiction between the local nature of the analysis conducted by Info-Gap and the severe nature of the uncertainty that Info-Gap is purported to deal with.

(iv) If the uncertainty is indeed severe – the estimate $\tilde{u}$ is poor and likely to be substantially wrong – it may be necessary to cover the total region of uncertainty with a GRID of points, or very small areas, as shown in Figure 13. This grid consists of 200 uniformly distributed circles whose total area is equal to the gray area of the local coverage scheme shown in Figure 12.

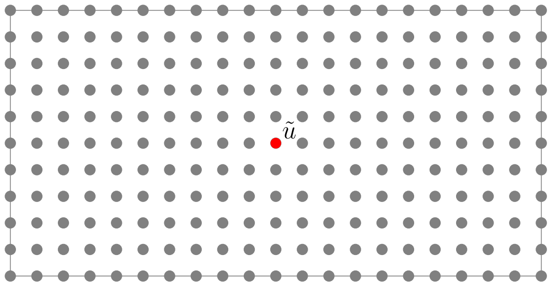


Figure 13: Grid-like non-local coverage of the region of uncertainty

(v) It is important to note that the term "local" does not imply that the area covered is by necessity small in comparison to the total area of the region of uncertainty. Local coverage may well be large, but it is still local in that it consists of an area (often a continuum) concentrated around the estimate $\tilde{u}$.

B.2 Does Info-Gap "simply sweep major risks under the carpet"?

Indeed, this is precisely what Info-Gap does!

What else can we say about a methodology that – under SEVERE uncertainty – does no more than conduct a worst-case analysis in the neighborhood of a POOR estimate that is likely to be SUBSTANTIALLY WRONG?

Those remaining unconvinced by this simple argument may wish to take another look at Figure 2 and Figure 5

B.3 Does Info-Gap generate robust decisions?

There is no reason to believe that this is so!

Info-Gap is crystal clear about the fundamental difficulty dogging decision-making under severe uncertainty: the parameters we have are *poor indications* of the true values they represent and are likely to be *substantially wrong* (Ben-Haim [2006, pp. 280-281]). Along the same lines,

> 1. Info-gap theory is useful precisely in those situations where our best models and data are highly uncertain, especially when the horizon of uncertainty is unknown. In contrast, if we have good understanding of the system then we don't need info-gap theory, and can use probability theory or even completely deterministic models. It is when we face severe Knightian uncertainty that we need info-gap theory. Ben-Haim [2007, p. 2]

Hence, throughout this discussion we assume exactly this:

Under severe uncertainty the estimate $\tilde{u}$ of Info-Gap's generic model is a poor indication of the true value of u and is likely to be substantially wrong.

Therefore, the fundamental question is this:

Given that $\tilde{u}$ is a poor indication of the true value of u and is likely to be substantially wrong, what can we claim about the *robustness* of decision q, denoted $\hat{\alpha}(q, r_c)$, as defined by Info-Gap?

Recall that $\hat{\alpha}(q, r_c)$ is defined by Info-Gap as follows:

$$\hat{\alpha}(q, r_c) := \max \left\{ \alpha \geq 0 : r_c \min_{u \in \mathcal{U}(\alpha, \tilde{u})} \leq R(q, u) \right\} \ , \ q \in \mathbb{Q} \tag{46}$$

To answer this question we apply the universal maxim that, *unless special measures are taken to remedy the situation*, if the values of the parameters of a model are wrong then the results generated by the model are equally likely to be wrong[5]. In the framework of the Info-Gap model, the picture then is this:

$$\xrightarrow{\text{Wrong } \tilde{u}} \boxed{\text{Info-Gap Model}} \xrightarrow{\text{Wrong } \hat{\alpha}(q, r_c)} \tag{47}$$

This in turn triggers the following logical progression:

· Info-Gap's decision model conducts its business in the immediate neighborhood of the estimate $\tilde{u}$, namely in $\mathcal{U}(\alpha, \tilde{u})$.

· Under severe uncertainty $\tilde{u}$ is a poor indication of the true value of u and is likely to be substantially wrong.

· Hence, the value of the robustness of decision $q \in \mathbb{Q}$ generated by Info-Gap, namely $\hat{\alpha}(q, r_c)$, is a *poor indication* of the true robustness of q and is likely to be *substantially wrong.*

· For the same reason the decision selected by the Info-Gap model, namely

$$q^*(r_c, \tilde{u}) := \arg \max_{q \in \mathbb{Q}} \hat{\alpha}(q, r_c) \tag{48}$$

is a *poor indication* of the true value of the optimal decision and is likely to be *substantially wrong.*

In short, there is *no reason to believe* that under severe uncertainty the solutions generated by Info-Gap are robust: there is no guarantee that a decision that is robust in the neighborhood of the poor estimate $\tilde{u}$ is robust in the neighborhood of the true value of u, or in any other neighborhood of the region of uncertainty.

Remark

The accepted wisdom in the Info-Gap literature is that one of Info-Gap's most attractive features is that it provides decision-makers with answers to questions of the following type:

· How wrong can my models and data be without jeopardizing the quality of the outcome?

· How wrong can my model and its parameters be before jeopardizing the quality of decisions made on the basis of this model?

· How wrong could my model be before I should change my decisions?

The trouble is that, as explained above, Info-Gap (being a Maximin model) can provide adequate answers to such questions ONLY if the estimate $\tilde{u}$ is a good indication of the true value of u. But in situations where $\tilde{u}$ is a poor indication of the true value of u and is likely to be substantially wrong (for which Info-Gap is designed), there is no ground whatsoever for the assertion that Info-Gap provides adequate answers to the above questions.

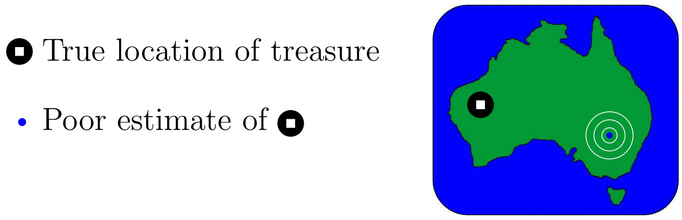


Figure 14: An Info-Gap Treasure Hunt

As I have been pointing out all along, the picture is this:

In other words, Info-Gap conducts its analysis somewhere in Victoria, whereas for all we know, the true (but unknown!) location of the treasure can be anywhere in Australia, say . . . West Australia, as shown in the picture.

There is another way to explain this misconception:

> The Info-Gap literature confuses two different contexts where the concept ROBUSTNESS is used to represent ability to withstand changes in the way a system operates:
>
> · Ability to cope with DEVIATIONS from a GIVEN FIXED POINT.
>
> · Ability to cope with SEVERE UNCERTAINTY.

Info-Gap deals with the ability of a system to withstand DEVIATIONS of u from a GIVEN fixed point $\tilde{u}$. This is totally different from dealing with the ability of a system to withstand SEVERE UNCERTAINTY in the true value of u (perform well over the region of uncertainty NOT KNOWING where the true value is located).

For the purposes of our discussion it suffices to say that the DEVIATION question addressed by Info-Gap has nothing to do with uncertainty, much less SEVERE uncertainty. Indeed, I can pose the three popular questions listed above in the context of a perfectly DETERMINISTIC system, as is the practice in all OR textbook dealing with SENSITIVITY ANALYSIS.

A more detailed discussion on the misconception in the Info-Gap literature regarding the difference between robustness against DEVIATION vs robustness against SEVERE UNCERTAINTY is forthcoming. Stay tuned.

B.4 Is Info-Gap a Worst-Case Analysis type of method?

Yes, it is!

Ben-Haim [2007, p. 2] claims that

> Info-gap theory is **not** a worst-case analysis, since there is no known worst case in an info-gap model of uncertainty.

Since we have already shown that Info-Gap's generic model is a Maximin model, it is manifest that Info-Gap is indeed a worst-case analysis par excellence. Nevertheless, . . .

The worst-case analysis conducted by Info-Gap is LOCAL in nature: it is conducted on each region of uncertainty $\mathcal{U}(\alpha, \tilde{u}), 0 \leq \alpha \leq \hat{\alpha}(r_c)$ (one at a time, so to speak). In fact, we

[5]This is a corollary of the famous universal *Garbage In* → *Garbage Out* Principle.

can formally state this as a classic Maximin operation as follows:

$$\hat{\alpha}(q,r_c) = \max_{\alpha\geq 0} \overbrace{\min_{u\in\mathcal{U}(\alpha,\tilde{u})} \alpha\cdot(r_c \preceq R(q,u))}^{\text{worst-case analysis}} \ , \ q\in\mathbb{Q} \tag{49}$$

That is, the robustness of decision q is equal to the largest value of α such that the *worst* u in $\mathcal{U}(\alpha,\tilde{u})$ satisfies the performance constraint (45). It is the min operation in (49) that carries out the worst-case analysis in a typical Maximin manner. And in the framework of the Info-Gap model itself, we have:

$$\hat{\alpha}(q,r_c) = \max\left\{\alpha\geq 0:\ r_c \leq \overbrace{\min_{u\in\mathcal{U}(\alpha,\tilde{u})} R(q,u)}^{\text{worst-case analysis}}\right\} \tag{50}$$

or in words,

$\hat{\alpha}(q,r_c) = \max\{\alpha :$ WORST element of u in $\mathcal{U}(\alpha,r_c)$ satisfies the performance constraint$\}$

Note that in the usual worst-case manner, the admissibility of the region of uncertainty $\mathcal{U}(\alpha,\tilde{u})$ associated with a given value of α, hence the admissibility of the value of α itself, is determined by the performance of the WORST element of this region, rather than say by the "average performance" on this region, or the "median" performance on this region, or "half the sum of the best performance and the worst performance" on this region, or the performance of the "most likely" element in this region, or . . .

If this is not a worst-case analysis par excellence, what is?!

B.5 Is there a worst case in an Info-Gap model of uncertainty?

Of course there is!

As explained in the preceding section, for any (q,α) pair for which $\hat{\alpha}(q,r_c)$ exists there is a worst case in $\mathcal{U}(\alpha,\tilde{u})$. That is,

$$u(q,\alpha) := \arg \min_{u\in\mathcal{U}(\alpha,\tilde{u})} R(q,u) \ , \ q\in\mathbb{Q}, \alpha\geq 0 \tag{51}$$

is the worst element of $\mathcal{U}(\alpha,\tilde{u})$ with respect to (q,α).

Remark:
The Info-Gap literature, eg Ben-Haim [2007], (erroneously) argues that there is no worst case in an Info-Gap model because α is unbounded above. This argument apparently construes "worst case" to mean "the worst element u in the TOTAL region of uncertainty $\mathfrak{U}$ with regard to a given decision q", namely,

$$u(q) := \arg \min_{u\in\mathfrak{U}} R(q,u) \ , \ q\in\mathbb{Q} \tag{52}$$

observing that if $\mathfrak{U}$ is unbounded then there is no guarantee that $u(q)$ exists.

However, it should be noted that this issue is totally irrelevant because according to Info-Gap the robustness of a decision q involves the REQUIREMENT CONSTRAINT $r_c \leq R(q,u), \forall u\in\mathcal{U}(\alpha,\tilde{u})$, so $\hat{\alpha}(q,r_c)$ may exist even if $u(q)$ does not.

For example, consider again the following simple case:

In this case we have $\mathfrak{U} = \mathbb{R}^2$ and therefore $u(q)$ does not exist. In other words, there is no worst u in $\mathfrak{U}$ with respect to $R(q,\cdot)$ because $R(q,\cdot)$ is not bounded below on $\mathfrak{U}$.

$$
\begin{aligned}
\tilde{u} &= (0,0)\\
\mathcal{U}(\alpha,\tilde{u}) &= \left\{u \in \mathbb{R}^2 : |u-\tilde{u}| \leq \alpha\right\} \ , \ \alpha \geq 0\\
\mathbb{Q} &= \{0,-1\}\\
R(q,u) &= q - u_1 - u_2\\
r_c &= -\sqrt{50}
\end{aligned}
$$

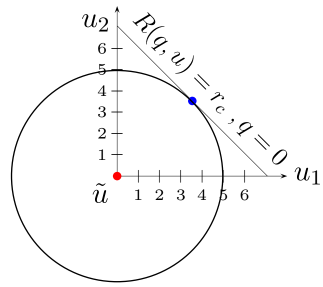


Figure 15: An Unbounded Info-Gap model

So what?

Insofar as the Info-Gap model is concerned, this is not an issue. For every (q,α) pair there is a worst u in $\mathcal{U}(\alpha,\tilde{u})$:

$$
\begin{aligned}
u(q,\alpha) :&= \arg \min_{u\in\mathcal{U}(\alpha,\tilde{u})} R(q,u) \ , \ q \in \mathbb{Q}, \alpha \geq 0 && (53)\\
&= \arg \min_{u\in\mathcal{U}(\alpha,\tilde{u})} q - u_1 - u_2 && (54)\\
&= \arg \min_{\substack{u\in\mathbb{R}^2\\ |u-\tilde{u}|\leq\alpha}} q - u_1 - u_2 && (55)\\
&= \left(\frac{\alpha}{\sqrt{2}}, \frac{\alpha}{\sqrt{2}}\right) && (56)
\end{aligned}
$$

Therefore, the worst value of $R(q,u)$ in $\mathcal{U}(\alpha,\tilde{u})$ is equal to

$$
\min_{u\in\mathcal{U}(\alpha,\tilde{u})} R(q,u) = R(q,u(q)) = q - \sqrt{2}\alpha \tag{57}
$$

and consequently the robustness of q is equal to the largest value of α such that $R(q,u(q)) \geq r_c$. This yields

$$
\hat{\alpha}(q,r_c) = \max\left\{\alpha \geq 0 : r_c \leq q - \sqrt{2}\alpha\right\} \tag{58}
$$

hence

$$
\hat{\alpha}(q,r_c) = \frac{q - r_c}{\sqrt{2}} \tag{59}
$$

and therefore for $r_c = -\sqrt{50}$ we obtain

$$
\begin{aligned}
\hat{\alpha}(q,r_c) &= \begin{cases} 5 & , \ q = 0\\ \dfrac{\sqrt{50}-1}{\sqrt{2}} & , \ q = -1 \end{cases} \ , \ q \in \mathbb{Q} = \{0,-1\} && (60)\\
&= \begin{cases} 5 & , \ q = 0\\ 5 - \dfrac{1}{\sqrt{2}} & , \ q = -1 \end{cases} && (61)
\end{aligned}
$$

Thus, the optimal decision is $q = 0$ and $\hat{\alpha}(r_c) = 5$.

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