Decision-Making Under Severe Uncertainty: An Australian, Operational Research Perspective

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This is a

Maths Classification G

presentation

Maths Classification MA +18

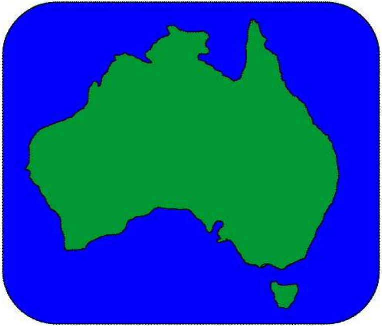
versions can be found at

www.ms.unimelb.edu.au/~moshe/

Programme

Inspiration

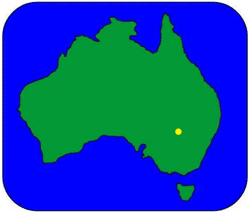
An old legend has it that an ancient treasure is hidden in an Asian-Pacific island.



You are the head of the treasure hunt. How would you plan the operation?

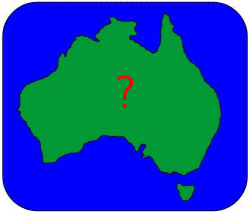
Inspiration

Terminology



Certainty

Risk



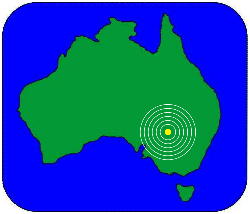
Severe Uncertainty

Main issue: location, location, location!

Inspiration

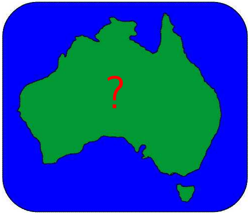
Certainty $\in$ Risk

Hence, the main issue is



Risk

vs

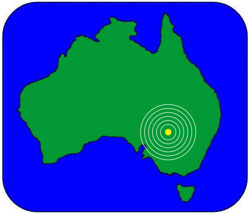


Severe Uncertainty

Inspiration

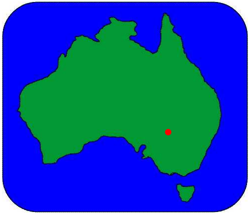
Severe Uncertainty

- The estimate is a poor indication of the true value of the parameter under consideration.
- The estimate is likely to be substantially wrong.
- The estimate is a wild guess.



Risk

vs



Severe Uncertainty

Examples, please!

Major Application Areas

- Risk finance
- Conservation planning
- Bio-security
- Homeland security

Examples, please!

AEDA Flyer . . .

. . . Which marine reserve design is most robust to the impacts of climate change, commercial fishing and coral bleaching? How many hectares of Mountain Ash forest can be logged before we lose Leadbeaters possum forever? What is the most efficient allocation of a finite budget for the surveillance and control of weed invasions in the Alpine National Park? What is the best strategy to quarantine healthy Tasmanian Devil populations? How much money should we spend on monitoring rare plants to be sure that we will detect a population crash in time to do something about it? Can we trust experts, and what should we do if experts disagree? Which species should be on the endangered species list, who gets to decide, and how should the list be used to allocate finite conservation resources? . . .

Examples, please!

AEDA Newsletter 8/10/07 . . .

. . . Billions of dollars are being spent across Australia on a range of environmental issues including weeds, feral pests, and revegetation. It's a big and ongoing investment yet we have little in the way of feedback informing us on the ecological return on this investment. The solution, of course, is to establish a monitoring program to provide this feedback. Unfortunately, effective monitoring is rarely implemented . . .

Examples

AEDA Newsletter 8/10/07 . . .

. . . the most fruitful approach to addressing the burning question of how to effectively monitor is via decision theory . . .

Who is Who?

$$$$$$ Major Players $$$$$$

- ABCRCEID: Australian Biosecurity CRC for Emerging Infectious Disease
- ACERA: Australian Centre of Excellence for Risk Analysis
- AEDA: Australian Environmental Decision Analysis Research Hub
- AMSI: Australian Mathematical Sciences Institute
- BRS: Bureau of Rural Sciences
- CSIRO: Marine and Atmospheric Research
- MASCOS: Mathematics and Statistics of Complex Systems

Australian chapter of SRA: Society for Risk Analysis

AEDA: 13 Areas of core research

* Theoretical ecology
* Conservation resource allocation
* Stochastic and spatial population models
* Decision theory
* Optimal monitoring systems
* Philosophy
* Field ecology
* Long-term monitoring data sets
* Statistical design
* Conservation planning
* Population biology
* Spatial modelling
* Population dynamics
* Bayesian decision theory
* Info-gap decision theory
* Urban ecology
* Social science and planning

ACERA

Research topics of particular interest to biosecurity risk analysis:

- ways in which expert opinion is gathered and used in risk assessment processes
- methods to support the interpretation of ongoing monitoring and surveillance activities
- the use of multi-criteria decision analysis as a tool to make decisions
- ways in which risk can be communicated to stakeholders.

Themes:

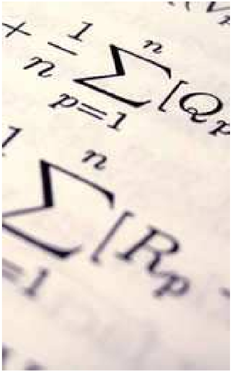
1. Biosecurity framework development
2. Eliciting judgements
3. Risk analysis methods
4. Surveillance and monitoring
5. Communication and decision making

ACERA

ACERA Call for Project Ideas

The Australian Centre of Excellence for Risk Analysis (ACERA) is seeking ideas for research projects.

We have a special interest in research that will improve Australia's biosecurity risk analysis. Biosecurity relates to the likelihood and consequences of the entry, establishment and spread of pests, diseases and pathogens. Our research is focused on biosecurity framework development, elicitation, risk analysis, monitoring and surveillance, and communication and decision making.



ACERA

acera
Australian Centre of Excellence for Risk Analysis

If you have a mathematical or statistical method, technique or approach that may improve biosecurity risk analysis, please complete the attached form and submit it to Dolla Boutros <dollab@unimelb.edu.au>, by no later than Friday 30 November 2007. If the idea has merit and matches ACERA's objectives, we will contact you to develop it further, working towards a full proposal that could be put to ACERA's Scientific Advisory Committee and DAFF in March 2008. In the first instance, we seek proposals that have budgets less than $50,000.

Successful projects would be scheduled to run between July 2008 and June 2009.

Concrete Example: Reserve Planning

Cells of a game reserve

1	2		$\cdots$					
					$j\,,\, p_{s,j}\,,\, c_j$			

$p_{s,j}$ = presence probability of species $s \in \mathbb{S}$ in cell $j \in \mathbb{J}$

c_j = cost of selecting cell j

The presence probabilities $\{p_{s,j}\}$ are subject to severe uncertainty.

What is the best (robust) subset of cells?

Concrete Example: Reserve Planning

Cells of a game reserve

1	2		$\cdots$					
					$j, p_{s,j}, c_j$			

Under certainty (True value of p is known):

$$\max_x \sum_{j\in\mathbb{J}}\sum_{s\in\mathbb{S}} x_j p_{s,j}$$

$$s.t. \quad \sum_{j\in\mathbb{J}} c_j x_j \le C$$

$$\sum_{j\in\mathbb{J}} x_j p_{s,j} \ge T_s \ , \ \forall s \in \mathbb{S}$$

$$x_j \in \{0,1\} \ , \ j \in \mathbb{J}$$

Under severe uncertainty: ????

Concrete Example: Reserve Planning

Cells of a game reserve

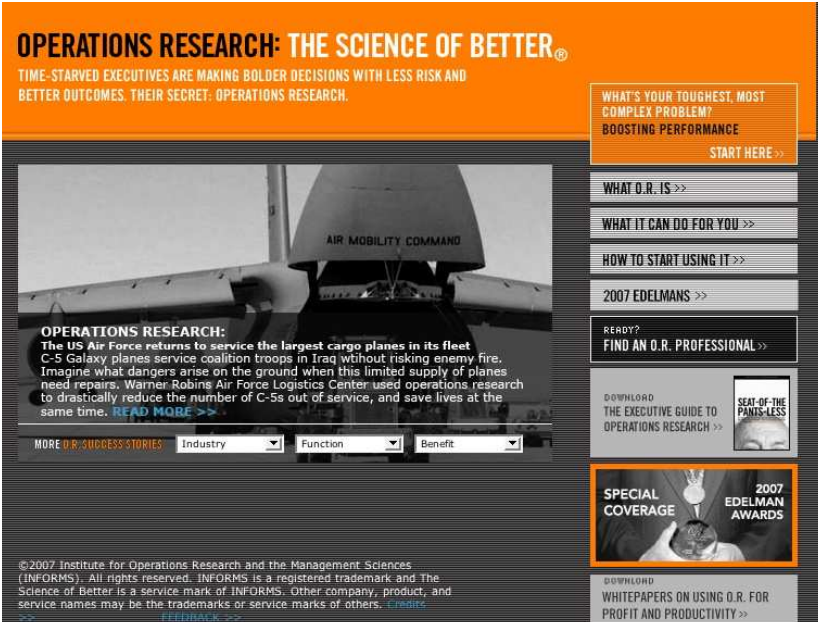
1	2		$\cdots$					
					$j\,,\,p_{s,j}\,,\,c_j$			

Under certainty (True value of p is known):

$$
\begin{aligned}
\max_{x} \ & \sum_{j\in\mathbb{J}}\sum_{s\in\mathbb{S}} x_j\, p_{s,j} \\
s.t.\quad & \sum_{j\in\mathbb{J}} c_j x_j \leq C \\
& \sum_{j\in\mathbb{J}} x_j\, p_{s,j} \geq T_s \ ,\ \forall s\in\mathbb{S} \\
& x_j \in \{0,1\}\ ,\ j\in\mathbb{J}
\end{aligned}
$$

Under severe uncertainty: $p_{s,j} = ?$

Perspective: OR



Perspective: OR

“ Time-starved executives are making bolder decisions with less risk and better outcomes ”

Perspective: OR

OPERATIONS RESEARCH: THE SCIENCE OF BETTER®

TIME-STARVED EXECUTIVES ARE MAKING BOLDER DECISIONS WITH LESS RISK AND BETTER OUTCOMES. THEIR SECRET: OPERATIONS RESEARCH.

What Operations Research Is

In a nutshell, operations research (O.R.) is the discipline of applying advanced analytical methods to help make better decisions.

By using techniques such as mathematical modeling to analyze complex situations, operations research gives executives the power to make more effective decisions and build more productive systems based on:

- More complete data
- Consideration of all available options
- Careful predictions of outcomes and estimates of risk
- The latest decision tools and techniques

Perspective: OR

OPERATIONS RESEARCH: THE SCIENCE OF BETTER®

TIME-STARVED EXECUTIVES ARE MAKING BOLDER DECISIONS WITH LESS RISK AND BETTER OUTCOMES. THEIR SECRET: OPERATIONS RESEARCH.

A uniquely powerful approach to decision making

You've probably seen dozens of articles and ads about solutions that claim to enhance your decision-making capabilities.

O.R. is unique. It is best of breed, employing highly developed methods practiced by specially trained professionals. It is powerful, using advanced tools and technologies to provide analytical power that no ordinary software or spreadsheet can deliver out of the box. And it is tailored to you, because an O.R. professional offers you the ability to define your specific challenge in ways that make the most of your data and uncover your most beneficial options.

Perspective: OR

To achieve these results, O.R. professionals draw upon the latest analytical technologies, including:

- **Simulation** Giving you the ability to try out approaches and test ideas for improvement
- **Optimization** Narrowing your choices to the very best when there are virtually innumerable feasible options and comparing them is difficult
- **Probability and Statistics** Helping you measure risk, mine data to find valuable connections and insights, test conclusions, and make reliable forecasts

Perspective: Personal

- $1945 < \cdots < 2003$
- 2003 - this morning
- Formal/informal contacts
  - Conservation biologists
  - Statisticians
  - Engineers
  - Mathematicians
  - Young
  - Mature
  - Tall
  - Short
  - others ...
- SRA 07, Hobart, August 21-22, 2007
- Book:

The Art and Science of Decision-Making Under Severe Uncertainty

Perspective

Key Issues

- Old vs New
- Optimizing vs Satisficing
- Robust vs Weak
- Uncertainty vs Variability
- Local vs Global
- Role of Mathematical Modeling
- Art of Mathematical Modeling
- Voodoo Decision-Making
- User-Interface

And the Winner is . . .

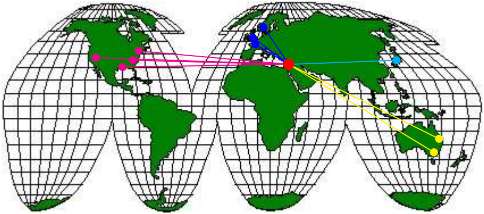
Planning for robust reserve networks using uncertainty analysis

Ecological Modelling, l99, pp. 115-124, 2006

. . . In summary, we recommend info-gap uncertainty analysis as a standard practice in computational reserve planning. The need for robust reserve plans may change the way biological data are interpreted. It also may change the way reserve selection results are evaluated, interpreted and communicated. Information-gap decision theory provides a standardized methodological framework in which implementing reserve selection uncertainty analyses is relatively straightforward. We believe that alternative planning methods that consider robustness to model and data error should be preferred whenever models are based on uncertain data, which is probably the case with nearly all data sets used in reserve planning . . .

And the Winner is . . .

Info-Gap Decision Theory



Profile

- Financial institutions
- Research centers
- Universities
- Government agencies

And the Winner is . . .

OR Reviews

First Edition (2001)

. . . Professor Yakov Ben-Haim has written a landmark book . . . His information- gap modeling approach to decision making under uncertainty constitutes a new and revolutionary approach for addressing tough decision problems when little information is available. . .

Prof. Keith Hipel
Dept. of Systems Design Engineering
University of Waterloo, Canada.

And the Winner is . . .

OR Reviews

First Edition (2001)

. . . The book presents a distinctive new theory of decision making under severe uncertainty . . . [T]his is a very comprehensive, focused and interesting book . . .

Prof. Daniel Sipper
Dept. of Industrial Engineering
Tel Aviv University, Israel.
Interfaces, 33(3), pp. 85-86, 2003.

And the Winner is . . .

Other Reviews

ARIA Newsletter, 7(1), 2002

. . . This book discusses an information-gap modeling approach to decision-making under uncertainty, constituting a new and revolutionary way to address tough decision problems. Info-gap decision theory is radically different from all current theories of decision under certainty. The difference originates in the modeling of uncertainty as an information gap to formulate decision algorithms, assess decision performance, identify and evaluate options, determine trade-offs and risks, and evaluate strategies for investigation. It has been written for decision analysts, including project management consultants, financial and economic analysts, engineers, and analysts in planning offices and public agencies, who provide quantitative support for the decision-making process in all areas in which systematic decisions are made . . .

And the Winner is . . .

Other Reviews

Second Edition (2006)

. . . Ben-Haim's book is widely in demand by those in my field because of its revolutionary strategy implications . . .

Cliford C. Dacso, MD, MBA
Distinguished Research Professor
University of Houston
John S. Dunn Sr. Research Chair in General Internal Medicine

My Recent Campaigns

From My Campaign Trails

- Method-Free Modeling
- Two-Envelope Paradox
- Maximin
- Voodoo Decision-Making
- Info-Gap

www.ms.unimelb.edu.au/∼moshe/

Findings . . .

Urgently Needed!

- Decision-making under severe uncertainty
- Robust optimization
- Pareto-Tradeoff
- Worst-Case Analysis
- Maximin
- Plain-text recipes
- User-Friendly software
- Sexy Buzzzzzzzzz words

Info-Gap: an outline, Ben-Haim [2001, 2006]

Generic Model

- $\mathcal{Q}$: decision space
- u : parameter of interest
- $\tilde{u}$: estimate of the true value of u
- $\mathcal{U}(\alpha, \tilde{u})$: region of uncertainty of size α centered at $u = \tilde{u}$
- $R(q, u)$: reward generated by decision q and parameter u
- r_c : critical level of reward

$$\hat{\alpha}(r_c) := \max_{q \in \mathcal{Q}} \max \left\{ \alpha : r_c \leq \min_{u \in \mathcal{U}(\alpha, \tilde{u})} R(q, u) \right\}$$

Region of Severe Uncertainty, $\mathfrak{U}$

Info-Gap: Myth and Facts

Myths

- Info-Gap is a new, revolutionary theory that is radically different from all current theories for decision making under uncertainty.
- Info-Gap is a methodology for robust decision-making under severe uncertainty.

Facts

Theorem 1, Sneidovich [2006]

Info-Gap's generic model is a simple instance of Wald's [1945] famous Maximin model.

Theorem 2, Sneidovich [2007]

Info-Gap's generic model does not deal with severe uncertainty, it simply and unceremoniously ignores it.

Myths and Facts

Info-Gap's format

Standard Maximin format

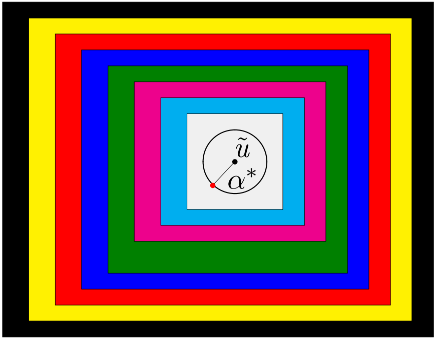
$$\max_{q\in\mathcal{Q}} \max \left\{ \alpha : r_c \leq \min_{u\in\mathcal{U}(\alpha,\tilde{u})} R(q,u) \right\} = \max_{q\in\mathcal{Q},\alpha\geq 0} \min_{u\in\mathcal{U}(\alpha,\tilde{u})} f(q,\alpha,u)$$

$$f(q,\alpha,u) := \begin{cases} \alpha & , \quad r_c \leq R(q,u) \\ -\infty & , \quad r_c > R(q,u) \end{cases} \quad , \quad u \in \mathcal{U}(\alpha,\tilde{u})$$

Myths and Facts

$$\hat{\alpha}(r_c) := \max_{q \in \mathcal{Q}} \max \left\{ \alpha : r_c \leq \min_{u \in \mathcal{U}(\alpha, \tilde{u})} R(q, u) \right\}$$

Fundamental Flaw



Key Issues

Key Issues

- Old vs New
- Optimizing vs Satisficing
- Uncertainty vs Variability
- Local vs Global robustness
- Art of Mathematical Modeling
- User-Interface

Key Issue: Old vs News

Reinvent the Wheel!

- Pareto-Tradeoff
- Maximin
- Robust Optimization

Key Issue: Optimizing vs Satisficing

Satisficing is better than Optimizing

- Optimal solutions have zero robustness
- OR = Science of Better! (Best?)
- Constraints vs Satisficing
- Simplicity vs Optimality
- Direct Optimization

Key Issue: Optimizing vs Satisficing

Theorem

Satisficing is better than optimizing.

Proof. Financial Times, August 25-27, 2005

…Optimization works in theory but risk management is better in practice. There is no scientific way to compute an optimal path for monetary policy …

Alan Greenspan

Key Issue: Optimizing vs Satisficing

Commentary, Blinder and Reis (2005, p. 13)

...But is this risk management paradigm something different from constrained optimization? Many academics think not. For example, Feldstein (2004, p. 42) interpreted Greenspan's risk management framework as solving (not literally, of course) a complex optimization problem:[14]

......

......

[14] This also seems to be Bernanke's (2004b) view.

Key Issue: Optimizing vs Satisficing

Myth

Satisficing is better than optimizing.

Fact

Any satisficing problem can be formulated as an (equivalent) optimization problem (Sniedovich [2006]).

Comments:

- Strictly and bluntly speaking, the assertion that satisficing is superior to optimizing is nonsensical.
- What is important is what you optimize and what you satisfice.
- This debate is counter-productive.

Key Issue: Optimizing vs Satisficing

Theorem (OR Folklore, see Sniedovich [2006])

Any satisficing problem can be expressed as an equivalent optimization problem.

Proof.

Let I denote the universal indicator function:

$$I_X(x) := \begin{cases} 1 & , \quad x \in X \\ 0 & , \quad x \notin X \end{cases} \tag{1}$$

Then clearly,

$$\underbrace{x \in X \subseteq X'}_{\text{Satisficing Problem}} \iff x = \arg \underbrace{\max_{x \in X'} I_X(x)}_{\text{Optimization Problem}} \tag{2}$$

$\square$

Myths and facts

Example

You win a game (AU\$5,000,000) if you select an action $q \in \mathbb{Q}$ such that $17 \leq \sigma(q) \leq 21$, where σ is a given real-valued function on $\mathbb{Q}$.

$$\text{Problem:} \quad \text{Find a } q \in \mathbb{Q} \text{ such that } 17 \leq \sigma(q) \leq 21$$

This is a typical satisficing model. Note that, in general, to win the game you do not necessarily optimize the score $\sigma(q)$ over $q \in \mathbb{Q}$. The following is an equivalent optimization model:

$$\max_{q \in \mathbb{Q}} \quad 5w(q) \tag{3}$$

$$w(q) := \begin{cases} 1 & , \quad 17 \leq \sigma(q) \leq 21 \\ 0 & , \quad \textit{otherwise} \end{cases} \tag{4}$$

Key Issue: Optimality vs Simplicity

Federal Reserve Board Paper No. 99 - 10 (January 8, 1999)

Simplicity Versus Optimality:
The Choice of Monetary Policy Rules
When Agent Must Learn

...One reason for the apparent preference for simple ad hoc rules over optimal rules might be the assumption of full information maintained in the computation of an optimal rule. Arguably this makes optimal control rules less robust to model specification errors...

Tetlow and Von Zur Meuhlen
Federal Reserve Board

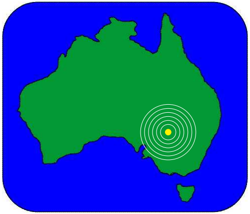
Key Issue: Variability vs Uncertainty

Robustness

- Ability to withstand deviations from a given value of the parameter of interest.

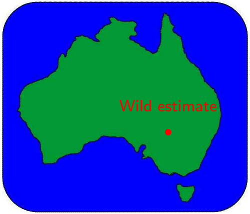
vs

- Ability to withstand severe uncertainty in the true value of the parameter of interest.



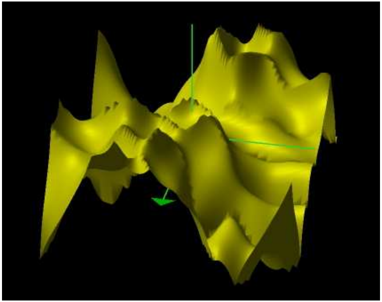
Deviations

vs



Severe Uncertainty

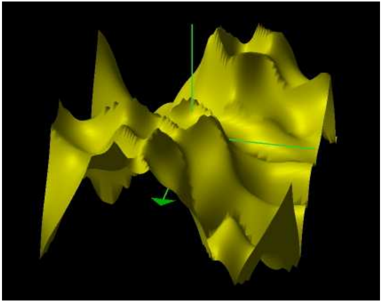
Key Issues: robustness against uncertainty

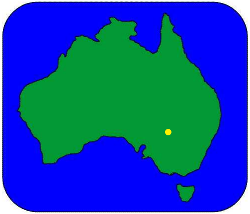


Local vs Global

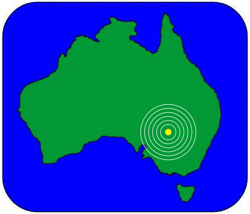
- Local vs Global optimization
- Voodoo decision-making
- "But . . . this is the best we have!?" syndrome

Key Issues: robustness against uncertainty

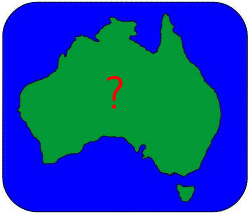




Certainty



Risk



Severe Uncertainty

Key Issue: Modeling

Art and Science of Mathematical Modeling

...We believe that decision theory and decision support is a lot more than software tools and protocols ...

The first task in any environmental management problem-solving exercise is posing the problem. A well-posed problem has several components: transparent quantifiable objectives, an explicit recognition of trade-offs and constraints ...and a suite of plausible management options. This happens all before we start thinking about solution methods...

AEDA Newsletter, Issue 11, October 19, 2007

Key Issue: Modeling

Art and Science of Mathematical Modeling

Info-Gap's format

Standard Maximin format

$$\max_{q\in\mathcal{Q},\alpha\geq 0}\left\{\alpha : r_c \leq \min_{u\in\mathcal{U}(\alpha,\tilde{u})} R(q,u)\right\} = \max_{q\in\mathcal{Q},\alpha\geq 0}\ \min_{u\in\mathcal{U}(\alpha,\tilde{u})} f(q,\alpha,u)$$

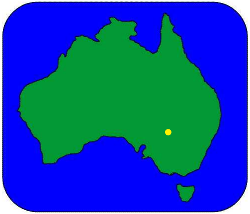
$$f(q,\alpha,u) := \begin{cases} \alpha & , \quad r_c \leq R(q,u) \\ -\infty & , \quad r_c > R(q,u) \end{cases} \quad , \quad u \in \mathcal{U}(\alpha,\tilde{u})$$

Key Issues: User Interface

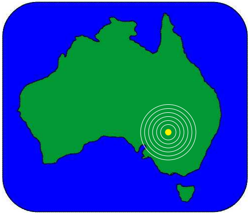
Marketing

- Relevant Examples
- Relevant Success stories
- Pictures/figures
- Fool-proof Recipes
- Friendly, inexpensive, software
- Sexy buzzzzzzzzZZZzzzzzzzz words

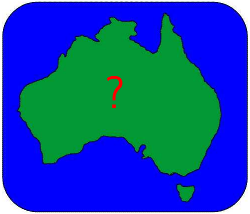
Conclusions



Certainty



Risk



Severe Uncertainty

Land of Plenty for OR !

- Plenty of opportunities
- Plenty of challenges
- Plenty of work
- Plenty of competitors . . .

Off the record

The Ten Natural Laws of Operations Analysis

Bob Bedow, *Interfaces* 7(3), p. 122, 1979

1. Ignore the problem and go immediately to the solution, that is where the profit lies.
2. There are no small problems only small budgets.
3. Names are control variables.
4. Clarity of presentation leads to aptness of critique.
5. Invention of the wheel is always on the direct path of a cost plus contract.
6. Undesirable results stem only from bad analysis.
7. It is better to extend an error than to admit to a mistake.
8. Progress is a function of the assumed reference system.
9. Rigorous solutions to assumed problems are easier to sell than assumed solutions to rigorous problems.
10. In desperation address the problem.

Of possible interest . . .

AMSI Summer School 2008
January 14 - February 15, 2008
Monash University

The Art and Science of Modeling, Analysing and Solving Decision-Making Problems

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