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Anatomy of a Misguided Maximin/Minimax Formulation of Info-Gap's Robustness Model

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Abstract

In the frameworks of classical decision theory and robust optimization, the quest for robustness against severe uncertainty is almost synonymous with the use of Wald's famous – some would say notorious – Maximin paradigm, or with one of its many variants (eg Savage's Minimax Regret model).

It is therefore puzzling, if not amazing, that both editions of the book on Info-Gap Decision Theory are utterly oblivious to this important paradigm. After all, Info-Gap was developed to provide a methodology that is designed especially for decision making under severe uncertainty.

What is more, not only is Info-Gap claimed to be a decision theory that is designed expressly for decision in the face of severe uncertainty, it is claimed to be a new non-probabilistic theory that is radically different from all current theories for decision under uncertainty.

These claims about Info-Gap are put forward in the Info-gap books without any substantiating analysis showing this to be the case. In other word, no analysis is provided to show how exactly is Info-Gap's robustness model different from the numerous instances of Wald's famous Maximin model.

Although this key methodological question was not discussed at the launch of Info-Gap, namely in the Info-gap books, it is addressed in later publications.

Thus, later articles on Info-gap address the relation between Info-Gap and Maximin but these articles make it their business to show that Info-Gap's robustness model is not a Maximin/Minimax model.

Now, I have shown formally (proved) on numerous occasions (articles, presentations, seminars, lectures, WIKIPEDIA discussions, etc) that this conclusion and the analysis on which it is based are totally erroneous. The fact is that Info-Gap's generic robustness model is a typical Maximin model, namely it is a simple, run-of-the-mill instance of Wald's Maximin model.

Not only have I repeatedly explained why the claims in the Info-Gap literature that Info-Gap's robustness model is not a Maximin/Minimax model are mistaken, I also formulated a

simple, fool-proof recipe that transforms any given instance of Info-Gap's robustness model into its Maximin counterpart.

Yet, amazing though it may sound, in a recent article Davidovitch and Ben-Haim (2008), repeat the same tired erroneous arguments. Similar arguments appear in Ben-Haim and Demertzis (2008).

So, the objective of this short paper is to explain, yet again, the conceptual and technical errors dogging the demonstration presumably showing that Info-Gap's robustness model is not a Maximin model.

In a nutshell: Davidovitch and Ben-Haim (2008) take themselves to demonstrate that Info-Gap's robustness model is not equivalent to a certain Minimax model. But the point here is that the comparison they make is between Info-Gap's robustness model and an ill-conceived Minimax model that they formulate ad hoc. Of course, the fact of the matter is that it is extremely easy to formulate a simple Maximin model that is totally equivalent to Info-Gap's robustness model. So, the inevitable conclusion is that Info-Gap's robustness model is a typical Maximin model in disguise.

Keywords: Maximin, info-gap, worst case analysis, robust optimization

Read Me First

In this paper I focus only on the erroneous arguments and conclusion in Davidovitch and Ben-Haim (2008, Section 4 Robust-Satisficing vs Minimax, p. 11) regarding the Info-Gap/Maximin connection.

I do not address here other failings of Info-Gap decision theory. These are discussed in Sniedovich (2007), WIKIPEDIA and other articles and presentations on my website `www.moshe-online.com`.

1 Introduction

Under consideration in this discussion is the nature of the relationship between two simple, abstract mathematical models: Wald's (1945, 1950) model and Info-Gap's robustness model (Ben-Haim 2001, 2006).

1.1 Maximin Model

The classic format of this famous model is as follows:

$$z^* := \max_{d \in D} \min_{s \in S(d)} f(d,s) \tag{1}$$

where:

- · D = decision space
- · $S(d)$ = set of states associated with decision d.
- · f = real-valued objective function on $D \times \mathbb{S}$, where $\mathbb{S} := \bigcup_{d \in D} S(d)$.

To this end, it is also convenient to use the equivalent, so called "Mathematical Programming", format (Kouvelis and Yu 1997), Sniedovich 2008a) of the Maximin model, recalling that

$$\begin{array}{ccc} \text{Classic Maximin Format} & & \text{MP Maximin Format} \\ \hline \max\limits_{d \in D} \min\limits_{s \in S(d)} f(d,s) & \equiv & \max\limits_{\substack{d \in D \\ z \in \mathbb{R}}} \{z : z \leq f(d,s), \forall s \in S(d)\} \end{array} \tag{2}$$

where $\mathbb{R}$ denotes the real line.

These models represent a game played by the Decision Maker and Nature. The former controls d and seeks to maximize $f(d,s)$ with respect to d over D, whereas the latter controls the value of s and endeavors to minimize $f(d,s)$ with respect to s over $S(d)$. Recall that this implies that the Decision Maker moves first and that Nature knows the value of d prior to responding to the decision maker's move.

1.2 Info-Gap's Robustness Model

In this discussion we use Davidovitch and Ben-Haim's (2008) formulation of the model, namely

$$\hat{\alpha}(L_c) := \max_{q \in \mathcal{Q}} \max \{\alpha \geq 0 : L_c \geq L(q,u), \forall u \in U(\alpha, \tilde{u})\} \tag{3}$$

where

- · L_c = given numeric value (critical level of loss)
- · $\mathcal{Q}$ = decision space
- · $\tilde{u}$ = estimate of the true value of a parameter $u \in \mathcal{U}$
- · L is a real-valued loss function on $\mathcal{Q} \times \mathcal{U}$
- · $U(\alpha, \tilde{u})$ = subset of $\mathcal{U}$, region of uncertainty of size $\alpha \geq 0$. It is assumed that

$$U(0, \tilde{u}) = \{\tilde{u}\} \tag{4}$$

$$U(\alpha, \tilde{u}) \subseteq U(\alpha + \varepsilon, \tilde{u}), \forall \alpha, \varepsilon \geq 0 \tag{5}$$

$$L_c \geq L(q, \tilde{u}) \ , \ \forall q \in \mathcal{Q} \tag{6}$$

The first point to note here is that the two max's in the robustness model can be combined, namely

$$\max_{q\in\mathcal{Q}} \max \{\alpha \geq 0 : L_c \geq L(q,u), \forall u \in U(\alpha,\tilde{u})\} \equiv \max_{\substack{q\in\mathcal{Q}\\ \alpha\geq 0}} \{\alpha : L_c \geq L(q,u), \forall u \in U(\alpha,\tilde{u})\} \tag{7}$$

This means that both conceptually and technically α can be viewed as a decision variable.

In the parlance of classical decision theory (Resnik 1987, French 1988), this model represents a two-players game: a Decision Maker vs. Nature. The decision maker controls the values of the two decision variables q and α, and Nature controls the value of the parameter u according to the rules of the game specified by the model.

Note that the $\forall$ sign in the model implies that given the pair (q,α), Nature's choice of u is restricted to the set $U(\alpha,\tilde{u})$. And the performance constraint $L_c \geq L(q,u), \forall u \in U(\alpha,\tilde{u})$ implies that Nature will aim to find a u in this set that violates this constraint. In short, as in the case of the Maximin model, Nature constitutes here an adversarial player.

1.3 The question

So, the question that we are raising here is as follows: what is the relationship between the following two simple, abstract mathematical models?

MP Maximin Model	Info-Gap's robustness model	
$\max_{\substack{d\in D\\ z\in\mathbb{R}}} \{z : z \leq f(d,s), \forall s \in S(d)\}$	$\max_{\substack{q\in\mathcal{Q}\\ \alpha\geq 0}} \{\alpha : L_c \geq L(q,u), \forall u \in U(\alpha,\tilde{u})\}$	(8)

In particular: Is it the case – as claimed in the Info-Gap literature – that Info-Gap's robustness model is radically different from Wald's Maximin/Minimax model?

2 The Theorem

It should be abundantly clear from the above analysis that Info-Gap's robustness model is a simple instance of Wald's Maximin model. To prove this formally all we need to show is that there are instances of the objects D, S and f such that the resulting instance of Wald's Maximin model is equivalent to Info-Gap's robustness model.

Theorem 1 *Info-Gap's robustness model is an instance of Wald's Maximin model. In other words, there exists a collection* $(D,\ S,\ f)$ *such that*

MP Maximin Model		*Info-Gap's robustness model*	
$\max_{\substack{d\in D\\ z\in\mathbb{R}}} \{z : z \leq f(d,s), \forall s \in S(d)\}$	$\equiv$	$\max_{\substack{q\in\mathcal{Q}\\ \alpha\geq 0}} \{\alpha : L_c \geq L(q,u), \forall u \in U(\alpha,\tilde{u})\}$	(9)

hence,

Classic Maximin Model		*Info-Gap's robustness model*	
$\max_{d\in D} \min_{s\in S(d)} f(d,s)$	$\equiv$	$\max_{q\in\mathcal{Q}} \max \{\alpha \geq 0 : L_c \geq L(q,u), \forall u \in U(\alpha,\tilde{u})\}$	(10)

The implication of this theorem is that Info-Gap's robustness model is a Maximin model in disguise. Any instance of Info-Gap's robustness model can be formulated as a Maximin model.

The added bonus of this proof is in its being constructive: it sets out a simple fool-proof recipe for the specification of a triplet (D,S,f) such that the resulting Maximin model is equivalent to Info-Gap's robustness model.

3 The proof

The task of figuring out how to set up the objects D and S is a relatively easy one. This is so because it is clear, almost by inspection, that the correspondence between the two models is via the following relationships:

$$d \longleftrightarrow (q, \alpha) \tag{11}$$

$$s \longleftrightarrow u \tag{12}$$

The reasoning behind this modeling observation is that in the context of Info-Gap's robustness model,

- The decision maker is in control of q and α.
- Uncertainty (Nature) controls u.

and in the context of the Maximin model,

- The decision maker is in control of d.
- Uncertainty (Nature) controls s.

So, in keeping with this conceptual modeling clue, we let

$$D = \mathcal{Q} \times \mathbb{R}_+ := [0, \infty) \tag{13}$$

$$S(d) = U(\alpha, \tilde{u}) \ , \ d = (q, \alpha) \in D \tag{14}$$

Substituting these instances in the Maximin model, we obtain the following:

MP Maximin Model	Info-Gap's robustness model	
$\max\limits_{\substack{q \in \mathcal{Q} \\ \alpha \geq 0 \\ z \in \mathbb{R}}} \{z : z \leq f(q, \alpha, u), \forall u \in U(\alpha, \tilde{u})\}$	$\max\limits_{\substack{q \in \mathcal{Q} \\ \alpha \geq 0}} \{\alpha : L_c \geq L(q, u), \forall u \in U(\alpha, \tilde{u})\}$	(15)

So all that is left to do is to determine what should be the appropriate instance of function f.

But this is spelled out clearly in (15): we aim that $f(q, \alpha, u)$ be equal to α whenever $L_c \geq L(q, u)$ and we want $f(q, \alpha, u)$ to represent a harsh penalty to prevent the choice of a triplet (q, α, u) such that $L_c < L(q, u)$.

This can be done in several ways using an appropriate penalty function. For example, we can set

$$g(q, \alpha, u) := \begin{cases} \alpha & , \ \ L_c \geq L(q, u) \\ -\infty & , \ \ L_c < L(q, u) \end{cases} \ , \ q \in \mathcal{Q}, \alpha \geq 0, u \in U(\alpha, \tilde{u}) \tag{16}$$

and let $f = g$, the convention being, as usual in a maximization problem that, $f(q, \alpha, u) = -\infty$ indicates that the choice of the decision variable $d = (q, \alpha)$ is prohibited.

Observe that by construction, for any triplet $(q \in \mathcal{Q}, \alpha \geq 0, u \in U(\alpha, \tilde{u}))$ we have

$$\alpha \leq g(q, \alpha, u) \longleftrightarrow L_c \geq L(q, u) \tag{17}$$

In fact, this choice implies that with no loss of generality, if there is an optimal solution, then the optimal value of z will be equal to the optimal value of α. Hence, we can set $z = \alpha$ in the Maximin model. This yields

MP Maximin Model		Info-Gap's robustness model	
$\max\limits_{\substack{q \in \mathcal{Q} \\ \alpha \geq 0}} \{\alpha : \alpha \leq g(q, \alpha, u), \forall u \in U(\alpha, \tilde{u})\}$	$\equiv$	$\max\limits_{\substack{q \in \mathcal{Q} \\ \alpha \geq 0}} \{\alpha : L_c \geq L(q, u), \forall u \in U(\alpha, \tilde{u})\}$	(18)

hence,

$$\begin{array}{ccc} \text{Classic Maximin Model} & & \text{Info-Gap's robustness model} \\ \hline \max\limits_{\substack{q\in\mathcal{Q}\\ \alpha\ge 0}} \min\limits_{u\in U(\alpha,\tilde{u})} g(q,\alpha,u) & \equiv & \max\limits_{\substack{q\in\mathcal{Q}\\ \alpha\ge 0}} \{\alpha : L_c \ge L(q,u), \forall u \in U(\alpha,\tilde{u})\} \end{array} \tag{19}$$

This concludes the formal proof. ★

It should be pointed out that other instances of f are also valid here. For example, consider the real-valued function $h = h(q,\alpha,u)$ defined as follows:

$$h(q,\alpha,u) := \alpha \cdot (L_c \succeq L(q,u)) \;\;,\;\; q\in\mathcal{Q}, \alpha\ge 0, u\in U(\alpha,\tilde{u}) \tag{20}$$

where

$$a \succeq b := \begin{cases} 1 & , \;\; a \ge b \\ 0 & , \;\; b < a \end{cases} \;\;,\;\; a,b\in\mathbb{R} \tag{21}$$

observing that by construction,

$$g(q,\alpha,u) = h(q,\alpha,u) = \alpha \tag{22}$$

for any triplet $q\in\mathcal{Q}, \alpha\ge 0, u\in U(\alpha,\tilde{u})$ such that $L_c \ge L(q,u)$.

In short,

Corollary 1 *Info-Gap's robustness model is equivalent to the following two instances of the Maximin model:*

$$\max_{\substack{q\in\mathcal{Q}\\ \alpha\ge 0}} \min_{u\in U(\alpha,\tilde{u})} g(q,\alpha,u) \equiv \max_{\substack{q\in\mathcal{Q}\\ \alpha\ge 0}} \min_{u\in U(\alpha,\tilde{u})} \alpha \cdot (L_c \succeq L(q,\alpha,u)) \tag{23}$$

Comments:

- The two instances of Wald's Maximin model that we examined above have a point and purpose: to state Info-Gap's robustness model in terms of the format required by Wald's Maximin model.
- As vividly brought out by the construction of these instances, this task can be quite tricky for the novice. It must therefore be carried out judiciously.
- From a mathematical modeling point of view the complicating factor here is the incorporation of Info-Gap's requirement constraint $L_c \ge L(q,u)$ in the objective function of the Maximin model, f. This may enjoin the use of a "penalty" function. Indeed, the only difference between the two instances we examined so far is the structure of the objective function of the Maximin model, namely g and h.
- Needless to say, if one's math modeling is done carelessly one may well end up with an instance of Wald's Maximin model that is not equivalent to Info-gap's robustness model.
- This remark may strike some readers as offensive in its patronizing tone.
- I should therefore like to reassure the reader that it is thoroughly justified in this case. Experience has shown that formulating Maximin models is not always a simple task even for those with considerable experience in this field. Immunity from serious conceptual and technical disasters is simply not guaranteed here.

With this in mind, let us now examine how the comparison Davidovitch and Ben-Haim (2008) make between the instance of Wald's Maximin model and Info-Gap's robustness model leads them to the erroneous conclusion that the latter is not a Maximin model.

4 Yet Another Minimax model . . .

In association with Info-Gap's robustness model consider the instance of Wald's Minimax model specified by the following choices of S, D and f:

$$D = \mathcal{Q} \tag{24}$$

$$S(q) = U(\alpha_m, \tilde{u}) \ , \ q \in \mathcal{Q} \tag{25}$$

$$f(q,u) = L(q,u) \ , \ q \in \mathcal{Q}, u \in U(\alpha_m, \tilde{u}) \tag{26}$$

where α_m is a given non-negative constant.

In other words, consider the following instance of the general Minimax model:

$$\min_{q \in \mathcal{Q}} \max_{u \in U(\alpha_m, \tilde{u})} L(q,u) \tag{27}$$

This is the instance chosen by Davidovitch and Ben-Haim (2008, p. 11) to demonstrate the relationship between Info-Gap's robustness model and Wald's classical Maximin/Minimax models. I shall refer to this model as DB.

As their objective is to demonstrate that Info-Gap's generic robustness model is not a Maximin model, it is instructive to have a clear visual picture of how the two models "look" when they are set off one against the other. This will enable us to see more clearly the similarities and differences between them:

DB Minimax Model	Info-Gap's robustness model	
$\min_{q \in \mathcal{Q}} \max_{u \in U(\alpha_m, \tilde{u})} L(q,u)$	$\max_{q \in \mathcal{Q}} \max \{\alpha \geq 0 : r_c \geq L(q,u), \forall u \in U(\alpha, \tilde{u})\}$	(28)
α_m = given fixed value		

So what is the verdict regarding this pair?

No detailed analysis is required here to immediately see that this instance of Wald's Minimax model cannot possibly be equivalent to Info-Gap's robustness model.

In fact, to even contemplate the possibility of considering it as a potential equivalent representation of Info-Gap's robustness model as a Minimax model is utterly misguided. So, one should reject this candidate outright on conceptual and technical grounds.

My long teaching experience propels me, however, to attach an explanation justifying why this conclusion is so obvious.

5 Comments on a Misguided Minimax Model

One rarely formulates a Maximin/Minimax model just for the sake of . . . formulating a Maximin/Minimax model. Invariably the model is constructed so as to provide a proper representation of a problem situation, be it abstract, concrete, real or imaginary.

For example, in the context of this discussion our aim is to formulate a Maximin/Minimax model that will give a proper representation of Info-Gap's robustness model. In other words, our objective is to formulate a specific Maximin/Minimax model that is equivalent to Info-Gap's robustness model.

But Davidovitch and Ben-Haim (2008) go out of their way to show that this is a mission impossible. It is not surprising therefore that the Minimax model that they eventually come up with for this purpose is not equivalent to Info-Gap's robustness model.

What is surprising though is the magnitude of the calamity arising from their modeling errors.

- Modeling Error # 1:
They are off to a bad start when a Minimax model rather than a Maximin model is postulated. Note that the proper choice in this matter is dictated by the decision-maker's objective. In the framework of Wald's models, this is represented by the "outer" optimization:

$$\begin{array}{cc} \text{Maximin Model} & \text{Minimax Model} \\ \overset{\text{DM}}{\max_{d\in D}}\ \overset{\text{NA}}{\min_{s\in S(d)}}\ f(d,s) & \overset{\text{DM}}{\min_{d\in D}}\ \overset{\text{NA}}{\max_{s\in S(d)}}\ f(d,s) \end{array} \tag{29}$$

where DM represents the decision maker and NA represents Nature (uncertainty).

Now, Info-Gap decision theory prides itself for maximizing robustness, so it is clear that the equivalent Wald model should be a Maximin model, not a Minimax model. Yet, Davidovitch and Ben-Haim (2008) decided to do it the other way round.

In short, what we established before even initiating the modeling effort itself, is that Info-Gap's robustness model calls for a comparison with a Maximin model of the form:

$$\max_{\substack{q\in\mathcal{Q}\\ \alpha\geq 0\\ ???}}\ \min_{\substack{u\in U(\alpha,\tilde{u})\\ ???}}\ f(q,\alpha,u) \tag{30}$$

where the ???'s represent, just in case, placeholders for "things yet to be determined". We also know that f should have the following property:

$$f(q,\alpha,u) = \begin{cases} \alpha & , \ L_c \geq L(q,u) \\ ??? & , \ L_c < L(q,u) \end{cases} \ , \ q\in\mathcal{Q}, \alpha\geq 0, u\in U(\alpha,\tilde{u}) \tag{31}$$

This is not a bad start. In fact, given the above, dealing with the ???s is a straightforward matter whereupon the formulation of the Maximin model is complete.

- Modeling Error # 2:
The next disaster follows immediately (the emphasis is mine):

> The technical difference between the two methods is the **fixed parameter**.
>
> When minimaxing, **we assume** that we know the horizon of uncertainty and that its value is α_m.
>
> Davidovitch and Ben-Haim (2008, p. 11)

This statement betrays a gross misapprehension of the formulation of Maximin/Minimax models.

There are no stipulations whatsoever in Maximin/Minimax modeling that certain objects that are not a priori fixed must be fixed by the modeler. Indeed, why should there be any such stipulations? Furthermore, how is it that Info-Gap's robustness model is not made subject to these stipulations while the Maximin/Minimax model is? By the same token, why is Info-Gap's robustness model allowed to treat α as a variable while the Maximin/Minimax model is not?

The **we assume** excuse/explanation/justification – or whatever else you wish to call it – should be interpreted as follows:

> The validity of the conclusions **we** reach at the end of **our** analysis depends crucially on this assumption. Obviously, **we** do not claim that **our** final results are valid if this assumption does not hold and/or it is not required.

Thus, by resorting to this assumption Davidovitch and Ben-Haim (2008) in effect pull the plug on their "demonstration".

In any case, this assumption is completely unwarranted: formally, from the decision-maker's perspective, α is a decision variable and the objective is to determine its largest feasible value subject to the performance requirement. As we have seen, this can be easily handled by a Maximin/Minimax model.

In short, the most that Davidovitch and Ben-Haim (2008) can claim given their assumptions is this:

> Subject to our (subjective) modeling assumptions we are unable to formulate a Minimax model that is equivalent to Info-Gap's robustness model. It may, however, be possible to accomplish this task by relaxing these assumptions.

As I have shown above, once these uncalled for assumptions are removed, the task can be handled easily by an experienced Maximin/Minimax modeler. Furthermore, the formulation of an equivalent model can be made perspicuous even to the novice.

To illustrate more vividly that assumptions of this kind can lead to wild conclusions, we now prove the impossible:

There are no kangaroos in Australia.

Proof.

1. When searching for kangaroos in Australia, we assume that the search area can be confined to downtown Melbourne.
2. It is well known that there are no kangaroos in downtown Melbourne.
3. Hence, there are no kangaroos in Australia.

QED

· Modeling Error # 3:
Another modeling disaster is that the crucial parameter L_c of Info-Gap's robustness model gets lost somewhere along the way to never reach its destination in the DB's Minimax model:

DB Minimax Model	Info-Gap's robustness model	
$\min\limits_{q\in\mathcal{Q}} \max\limits_{u\in U(\alpha_m,\tilde{u})} L(q,u)$ $\alpha_m =$ given fixed value	$\max\limits_{q\in\mathcal{Q}} \max \{\alpha \geq 0 : L_c \geq L(q,u), \forall u \in U(\alpha,\tilde{u})\}$	(32)

Where is L_c in the DB Minimax Model?!?

How can a model properly represent Info-Gap's robustness model if it does not include the key parameter L_c? It can't, hence it is astounding that such a model is given serious considerations, let alone is used to provide the foundation for a claim that the extremely flexible Maximin/Minimax model cannot cope with the simple modeling job under consideration.

In summary, this brief examination indicates clearly that Davidovitch and Ben-Haim (2008) have serious misconceptions about the modeling capabilities of the Mighty Maximin/Minimax paradigm.

6 Remark

However grave the modeling errors in Davidovitch and Ben-Haim (2008), some leniency may perhaps be shown to their project, given our admission that immunity from error is not guaranteed in the modeling of the Maximin/Minimax paradigm.

However, no such leniency can possibly be shown to their negligence to consider in their analysis an instance of Wald's Maximin model that is equivalent to Info-Gap's robustness model. Indeed, my harshest criticism is reserved for this aspect of their exposition.

Davidotvich and Ben-Haim (2008) knowingly exclude from the discussion a Maximin model that contradicts their basic thesis (see the Wikipedia[1] article and discussion page on Info-Gap decision theory). What are we to make of their comparison given that they have at their disposal a Maximin model that is equivalent to Info-Gap's robustness model?

7 Conclusion

Wald's Maximin/Minimax model provides an extremely powerful and flexible modeling environment for decision-making under severe uncertainty (see the paper *The Mighty Maximin* (Sniedovich 2008)). It plays a central role in classical decision theory (Resnik 1987, French 1988) and robust optimization (Kouvelis and Yu 1997, Ben-Tal et al 2006). It is therefore not surprising at all that this paradigm easily subsumes Info-Gap's robustness model as a special case.

The trouble with Davidotvich and Ben-Haim (2008) is not only in its conclusion being woefully wrong. It is primarily in the manner in which the attempt to formulate a Maximin/Minimax representation of Info-Gap's robustness model is carried out.

The following summary speaks for itself.

$$
\begin{array}{c|c}
\multicolumn{2}{c}{\text{Info-Gap's Robustness Model}} \\
\multicolumn{2}{c}{\max_{q\in\mathcal{Q}} \max \ \{\alpha \geq 0 : L_c \geq L(q,u), \forall u \in U(\alpha,\tilde{u})\}} \\
\hline
\text{Equivalent Maximin Model} & \text{Misguided Minimax Model} \\
\max_{\substack{q\in\mathcal{Q}\\ \alpha\geq 0}} \ \min_{u\in U(\alpha,\tilde{u})} \ \alpha\cdot(L_c \succeq L(q,u)) & \min_{q\in\mathcal{Q}} \ \max_{u\in U(\alpha_m,\tilde{u})} \ L(q,u) \\
a \succeq b := \begin{cases} 1 & , \ a \geq b \\ 0 & , \ a < b \end{cases} & \alpha_m = \ \text{some fixed value} \\
\hline
\text{Sniedovich (2007)} & \text{Davidovitch and Ben-Haim (2008)}
\end{array}
\tag{33}
$$

Viva Maximin!

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