Black Swans, New Nostradamuses, Voodoo Decision Theories and the Science of Decision-Making in the Face of Severe Uncertainty

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Programme

- How do you make responsible (robust?) decisions in the face of severe uncertainty?
  - Severe Uncertainty
  - Decision Models
  - Voodoo Decision Theories
  - Robustness Against Severe Uncertainty
  - Local Robustness
  - Case Study: The Campaign
  - Conclusions: OR Perspective

This is a

Maths Classification G

presentation.

Maths Classification MA +18

versions can be found at

www.moshe-online.com

Motivation

Opening paragraph of an on-line article on the FloodRiskNet website (UK):

Hall and Ben-Haim, 2007, p. 1

Making Responsible Decisions (When it Seems that You Can't)

Engineering Design and Strategic Planning Under Severe Uncertainty

What happens when the uncertainties facing a decision maker are so severe that the assumptions in conventional methods based on probabilistic decision analysis are untenable?Jim Hall and Yakov Ben-Haim describe how the challenges of really severe uncertainties in domains as diverse as climate change, protection against terrorism and financial markets are stimulating the development of quantified theories of robust decision making.

Severe Uncertainty

All around us!

- Climate
- Environment
- Finance
- Economy
- Politics
- Personal life!

Severe Uncertainty

Example

bio-security homeland-security

A Simple Tutorial Problem

Good morning Sir/Madam:

I left on your doorstep four envelopes. Each contains a sum of money. You are welcome to open any one of these envelopes and keep the money you find there.

Please note that as soon as you open an envelope, the other three will automatically self-destruct, so think carefully about which of these envelopes you should open.

To help you decide what you should do, I printed on each envelope the possible values of the amount of money (in Australian dollars) you may find in it. The amount that is actually there is equal to one of these figures.

Unfortunately the entire project is under severe uncertainty so I cannot tell you more than this.

Good luck!

Joe.

So What Do You do?

Example

Envelope	*Possible Amount (Australian dollars)*
$E1$	$20, 10, 300, 786$
$E2$	$2, 40000, 102349, 5000000, 99999999, 56435432$
$E3$	$201, 202$
$E4$	200

Vote!

Modeling and Solution

- What is a decision problem under severe uncertainty?
- How do we model such decision problems?
- How do we solve such decision problems?

Decision Tables

Think about your problem as a table, where

- rows represents decisions
- columns represent the relevant possible states of nature
- entries represent the associated payoffs/rewards/costs

Example

Env	*Possible Amount ($AU)*				
$E1$	20	10	300	786	
$E2$	2	4000000	102349	500000000	56435432
$E3$	201	202			
$E4$	200				

Classification of Uncertainty

Classical decision theory distinguishes between three levels of uncertainty regarding the state of nature, namely

- Certainty
- Risk
- Strict Uncertainty

Terminology:

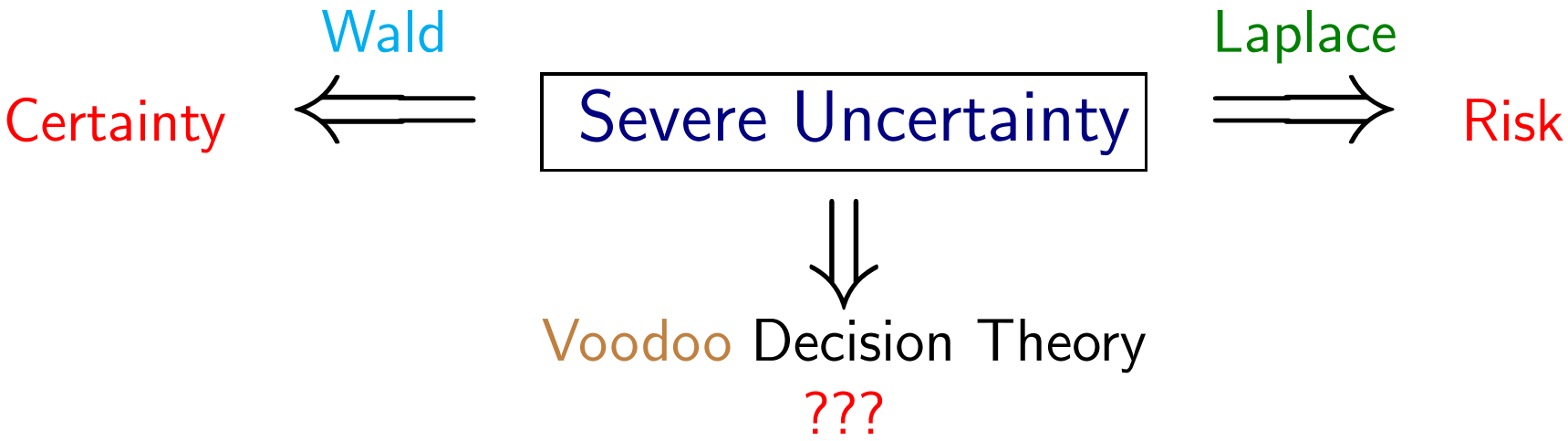
$$\begin{aligned} \text{Strict Uncertainty} &\equiv \text{Severe Uncertainty} \\ &\equiv \text{Ignorance} \\ &\equiv \text{True Uncertainty} \\ &\equiv \text{Knightian Uncertainty} \\ &\equiv \text{Deep} \\ &\equiv \text{Extreme} \end{aligned}$$

Severe Uncertainty

Classical decision theory offers two basic principles for dealing with severe uncertainty, namely

- Laplace's Principle (1825)
- Wald's Principle (1939)

Conceptually:



Laplace vs Wald

Example

Env	*Possible Amount ($AU)*					*Laplace*	*Wald*
$E1$	20	10	300	786		279	10
$E2$	2	4000	10234	50000	56435	24134.2	2
$E3$	201	202				201.5	201
$E4$	200					200	200

Cygnus atratus

Black Swans

Nassim Nicholas Taleb

- Fooled by Randomness, Random House, 2005.
- The Black Swan: The Impact of the Highly Improbable, Random House, 2007.

Black Swans

Taleb, 2007

A Black Swan is a highly improbable event with three characteristics:

- It is totally unpredictable.
- Its impact is massive.
- It is amenable to explanation, after the fact, so that in retrospect it appears predictable, not random.

- Their distinctive characteristics (as "rare events") put them outside the purview of formal mathematical treatment.
- Taleb's criticism of methods and models that are the staple fare of the OR curriculum (e.g. classic portfolio analysis) has no doubt infuriated many OR specialists.

Taleb's recipe for the treatment of Black Swans

1. What is fragile should break early while it is still small.
2. No socialisation of losses and privatisation of gains.
3. People who were driving a school bus blindfolded (and crashed it) should never be given a new bus.
4. Do not let someone making an "incentive" bonus manage a nuclear plant — or your financial risks.
5. Counter-balance complexity with simplicity.
6. Do not give children sticks of dynamite, even if they come with a warning.
7. Only Ponzi schemes should depend on confidence. Governments should never need to "restore confidence".
8. Do not give an addict more drugs if he has withdrawal pains.
9. Citizens should not depend on financial assets or fallible "expert" advice for their retirement.
10. Make an omelette with the broken eggs.

Black Swans

$64K Question

Can OR offer tools that are capable of coping with Black Swans?

Michel de Nostradamus (1503-1566)

Occupation: Apothecary, author, translator, astrological consultant

New Nostradamuses

Associated Press, March 4, 2009:

President Barack Obama will order martial law this year, the U.S. will split into six rump-states before 2011, and Russia and China will become the backbones of a new world order

http://www.foxnews.com/story/0,2933,504384,00.html

Source: Igor Panarin

Dean of the Russian Foreign Ministry diplomatic academy, a regular on Russia's state-controlled TV channels, a former spokesman for Russia's Federal Space Agency, and reportedly an ex-KGB analyst.

New Nostradamuses

Igor Panarin

New Nostradamuses

Prof. Bruce Bueno de Mesquita

New Nostradamuses

Bruce Bueno de Mesquita

Bruce Bueno de Mesquita is a political scientist, professor at New York University, and senior fellow at the Hoover Institution. He specializes in international relations, foreign policy, and nation building. He is also one of the authors of the selectorate theory.

He has founded a company Decision Insights (www.diiusa.com) that specializes in making political and foreign-policy forecasts using a computer model based on game theory and rational choice theory. He is also the director of New York University's Alexander Hamilton Center for Political Economy.

In a recent book, Bueno de Mesquita (2009) explains how we can see and shape the future using the logic of brazen self-interest within a game theoretic framework.

New Nostradamuses

www.diiusa.com/about.html

Decision Insights, with offices in New York and Washington, is a unique information company that possesses the most accurate decision-making and problem-solving system available in the world today.

A system that has successfully analyzed thousands of sensitive issues for government and business obtaining a verifiable accuracy rate exceeding 90% plus.

New Nostradamuses

Bueno de Mesquita's new book

Bueno de Mesquita, B., *The Predictioneer's Game: Using the Logic of Brazen Self-Interest to See and Shape the Future.* Random House, 2009.

Where is the beef?

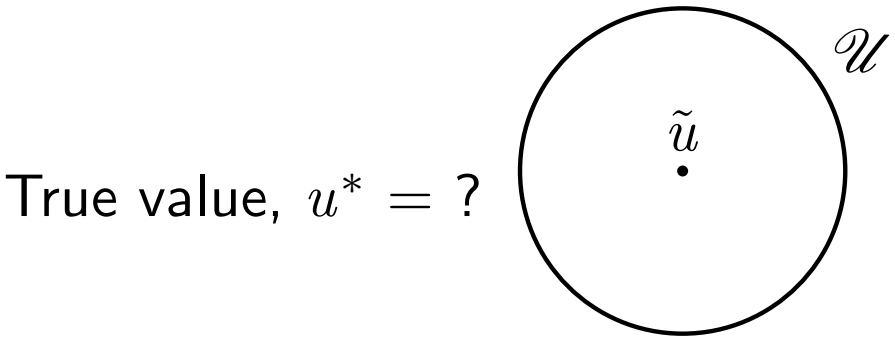
Bueno De Mesquita does not provide the details of the models he uses for these predictions. He claims better than 90% success rate!

Critique of Bueno de Mesquita's work

See
http://decision-making.moshe-online.com/criticism_of_bueno_de_mesquita.html

A simple model of severe uncertainty

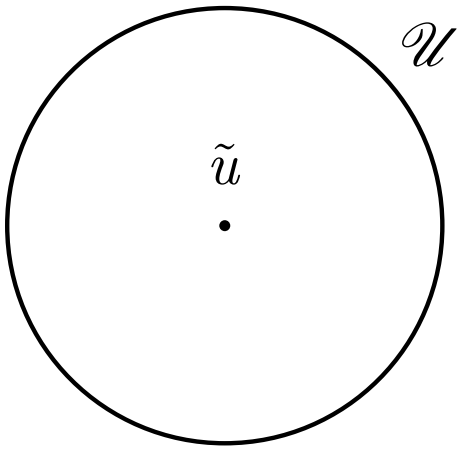
- Uncertainty space, $\mathscr{U}$.
  This is the set of possible/probable values of a parameter of interest, u. Given that the uncertainty is severe, this set can be vast.
- $u^* \in \mathscr{U}$: is the "true" value of u.
  As this value is subject to severe uncertainty, *all* we know about it is that it is an element of $\mathscr{U}$.
- $\tilde{u} \in \mathscr{U}$: a point estimate of u^*.
  Given that the uncertainty is severe, we assume that $\hat{u}$ is a poor indication of u^*, meaning that it is likely to be substantially wrong.



A simple model of severe uncertainty

Characteristics

- The uncertainty space $\mathscr{U}$ can be vast.
- The point estimate $\tilde{u}$ is of extremely poor quality.
- The model is devoid of any likelihood structure.



True value, $u^* = ?$

Decision Model

Conceptual framework

Uncertainty free Problem	Parametric Problem	Robust Counterpart Problem
Problem P	Problem P(u)	Problem P$(\mathscr{U}, \hat{u})$
$x \in X, f(x) \in \mathcal{F}$	$x \in X(u), f(x;u) \in \mathcal{F}$	?

? = ?

Robust Counterpart Problem

Select a decision $x \in X$ such that $f(x;u)$ performs well over $u \in \mathscr{U}$ given the point estimate $\tilde{u} \in \mathscr{U}$.

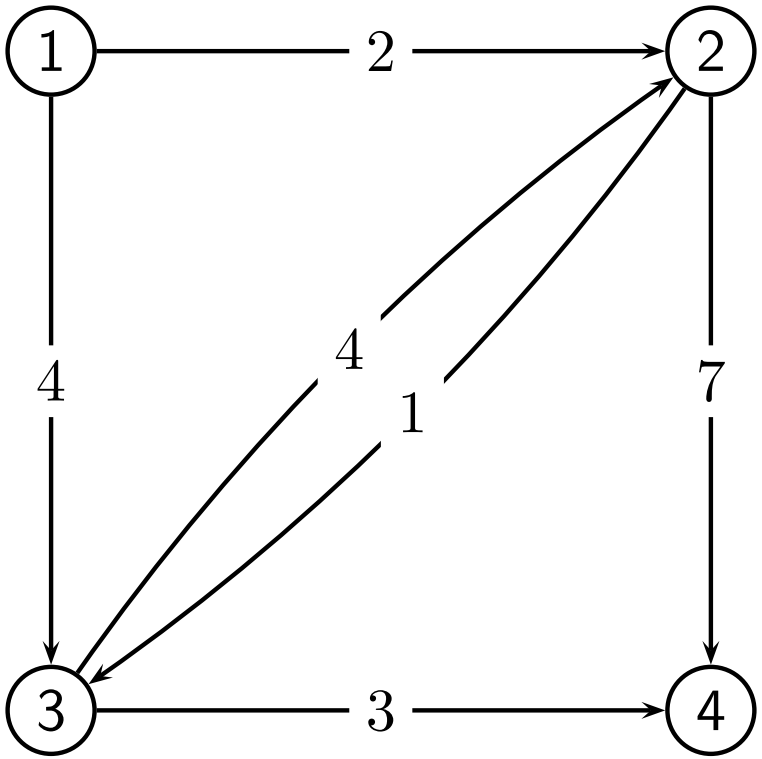
performs well $=$?

Robust Counterpart

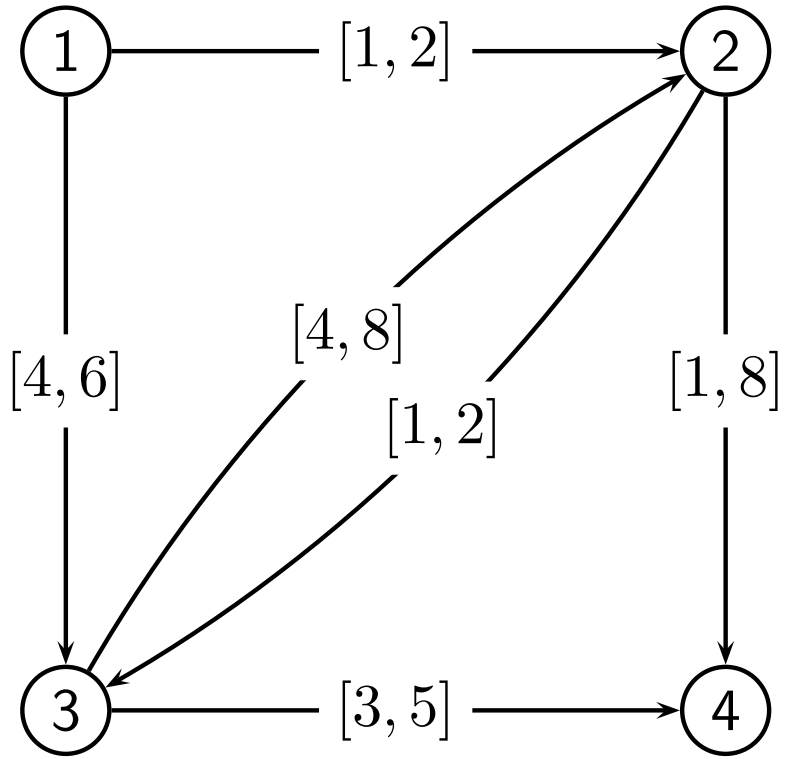
Simple Example

Shortest path problem with variable arc lengths

"Conventional version"



"Robust version"



Voodoo Decision Theories

Good company!

- Voodoo economics
- Voodoo science
- Voodoo statistics
- Voodoo mathematics
- Voodoo decision-making

Definition

A voodoo decision theory is a decision theory that lacks sufficient evidence or proof, is based on utterly unrealistic and/or contradictory assumptions, spurious correlations, and so on.

Voodoo Decision Theories

The behavior of Kropotkin's cooperators is something like that of decision makers using the Jeffrey expected utility model in the Max and Moritz situation. Are ground squirrels and vampires using voodoo decision theory?

Skyrms (1996, p. 51)

Voodoo Decision Theories

The principles that are typically contravened by a Voodoo decision theory:

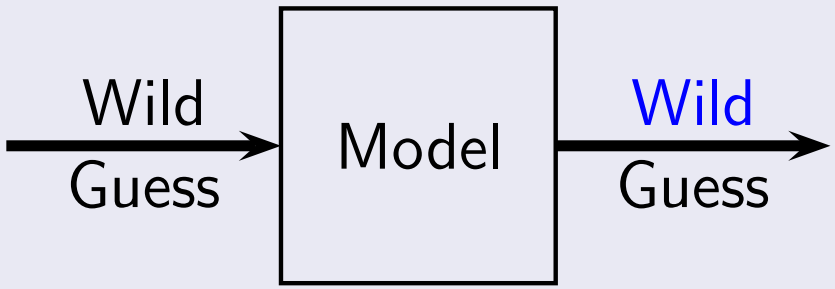
Universally accepted Maxims

- Garbage In — Garbage Out.
- Results are only as good as the estimate on which they are based.
- Principium Contradictionis: a theory should not contradict . . . itself!

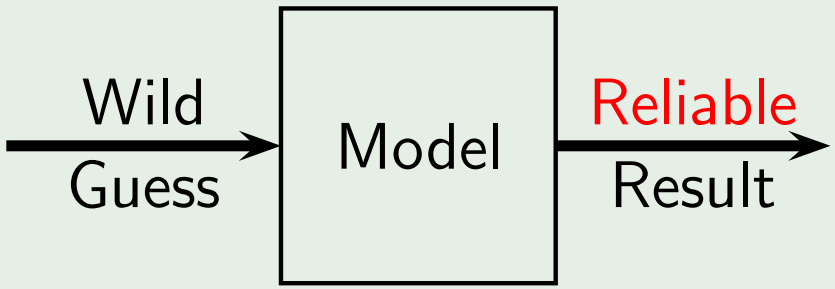
Voodoo Decision Theories

Advantage of Voodoo Decision Theories

Conventional Scientific Theory



Voodoo Theory



Voodoo Decision Theories

Common Recipe

The $1 \star 2 \star 3$ Voodoo Recipe

1. Pick a wild guess of the true value of the parameter of interest.
2. Ignore the severity of the uncertainty, the vastness of the uncertainty space, and the poor quality of the wild guess.
3. Conduct an analysis in the immediate neighborhood of this wild guess so as to seek out a decision that is robust in this neighborhood.

Warning!

It is not always easy to recognize a Voodoo decision theory in its natural habitat.

Robustness Against Severe Uncertainty

The dominating approach in decision-making under severe uncertainty is

Worst-case analysis

Hope for the best, plan for the worst!

Early version

The gods to-day stand friendly, that we may,
Lovers of peace, lead on our days to age!
But, since the affairs of men rests still incertain,
Let's reason with the worst that may befall.

Julius Caesar, Act 5, Scene 1
William Shakespeare (1564-1616)

Assumption

Nature (uncertainty) is playing against us!

Wald's Maximin model (1939, 1945, 1950)

Decision Rule

Rank decisions based on their worst possible outcomes: select the decision the worst outcome of which is at least as good as the worst outcome of the other available decisions.

Math Formulation

Classic format		MP format
$\max_{x \in X} \min_{s \in S(x)} g(x,s)$	$\equiv$	$\max_{x \in X, v \in \mathbb{R}} \{v : v \leq g(x,s), \forall s \in S(x)\}$

$$\overset{\text{DM}}{\max_{x \in X}} \quad \overset{\text{Nature}}{\min_{s \in S(x)}} \quad g(x, s)$$

Wald's Maximin model (1939, 1945, 1950)

Robustness wrt Constraints

MP Format:

$$\max_{x \in X, v \in \mathbb{R}} \{v : v \leq g(x,s), c(x,s) \leq 0, \forall s \in S(x)\}$$

Classic Format:

$$\max_{x \in X} \min_{s \in S(x)} \varphi(x,s)$$

where

$$\varphi(x,s) := \begin{cases} g(x,s) & , \quad c(x,s) \leq 0 \\ -\infty & , \quad c(x,s) > 0 \end{cases}$$

Wald's Maximin model (1939, 1945, 1950)

Moderating the Conservatism of the Worst-Case Approach

Savage's Minimax Regret (1951, 1954)

Minimize the maximum regret: regret = difference between an outcome and the best (over all decisions) outcome pertaining to a given state.

Starr's Domain Criterion (1963, 1966)

Rank decisions on the basis of the "size" of the subset of the uncertainty space over which they perform well.

Globalized Robustness (Ben-Tal et al 2006, 2009)

Allows controlled violations of "desired" levels of performance.

Wald's Maximin model (1939, 1945, 1950)

Globalized Robustness (Ben-Tal et al 2006, 2009)

Consider the parametric optimization problem

$$\max_{x \in X} \; g(x) \;\; \textit{subject to} \;\; c(x,u) \leq 0$$

where c is a real-valued function on $X \times \mathscr{U}$.

If the constraint $c(d,u) \leq 0$ is "hard" we may have to consider the following *robust-counterpart* version of the problem:

$$z^* := \max_{x \in X} \; g(x) \;\; \textit{subject to} \;\; c(x,u) \leq 0, \forall u \in \mathscr{U}$$

But . . . this could be too demanding (conservative).

Wald's Maximin model (1939, 1945, 1950)

Globalized Robustness (Ben-Tal et al 2006, 2009)

$$z^* := \max_{x \in X} \; g(x) \;\; \textit{subject to} \;\; c(x,u) \leq 0, \forall u \in \mathscr{U}$$

But . . . this could be too demanding (conservative). So consider instead

$$\max_{x \in X} \; g(x) \;\; \textit{subject to} \;\; c(x,u) \leq \beta \cdot dist(u, \mathcal{N}), \forall u \in \mathscr{U}$$

where $\beta \geq 0$, and

$$dist(u, \mathcal{N}) = \text{distance from } u \text{ to } \mathcal{N} \subset \mathscr{U}$$

so that $dist(u, \mathcal{N}) = 0, \forall u \in \mathcal{N}$, eg.

$$dist(u, \mathcal{N}) := \inf_{v \in \mathcal{N}} ||v - u||$$

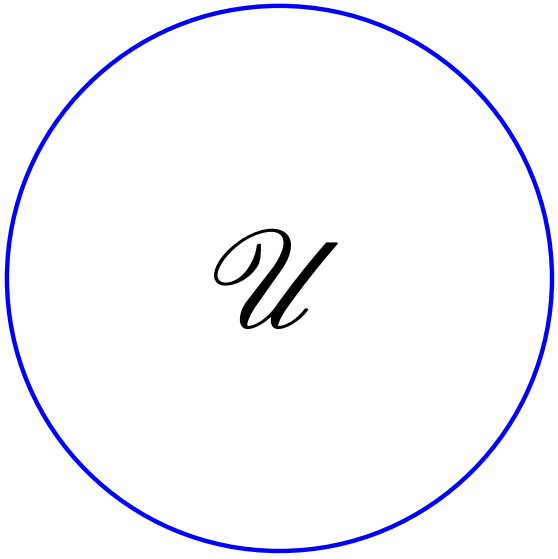
Local Robustness

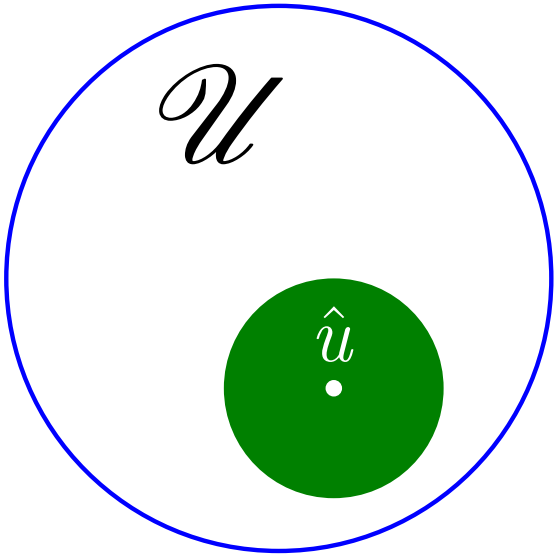
Global Robustness

Find a decision $d \in D$ that performs well — relative to other decisions — over the entire uncertainty space $\mathscr{U}$.

Local Robustness

Find a decision $d \in D$ that performs well — relative to other decisions — in the neighborhood of a given point $\hat{u} \in \mathscr{U}$.





Local Robustness

Radius of Stability (1960s)

- A very popular measure of local robustness.
- Used extensively in control theory, parametric programming, etc.
- Formal definition:

$$\rho(d, \hat{u}) := \max\{\alpha \geq 0 : c(d, u) \in C, \forall u \in \mathscr{B}(\alpha, \hat{u})\}$$

where

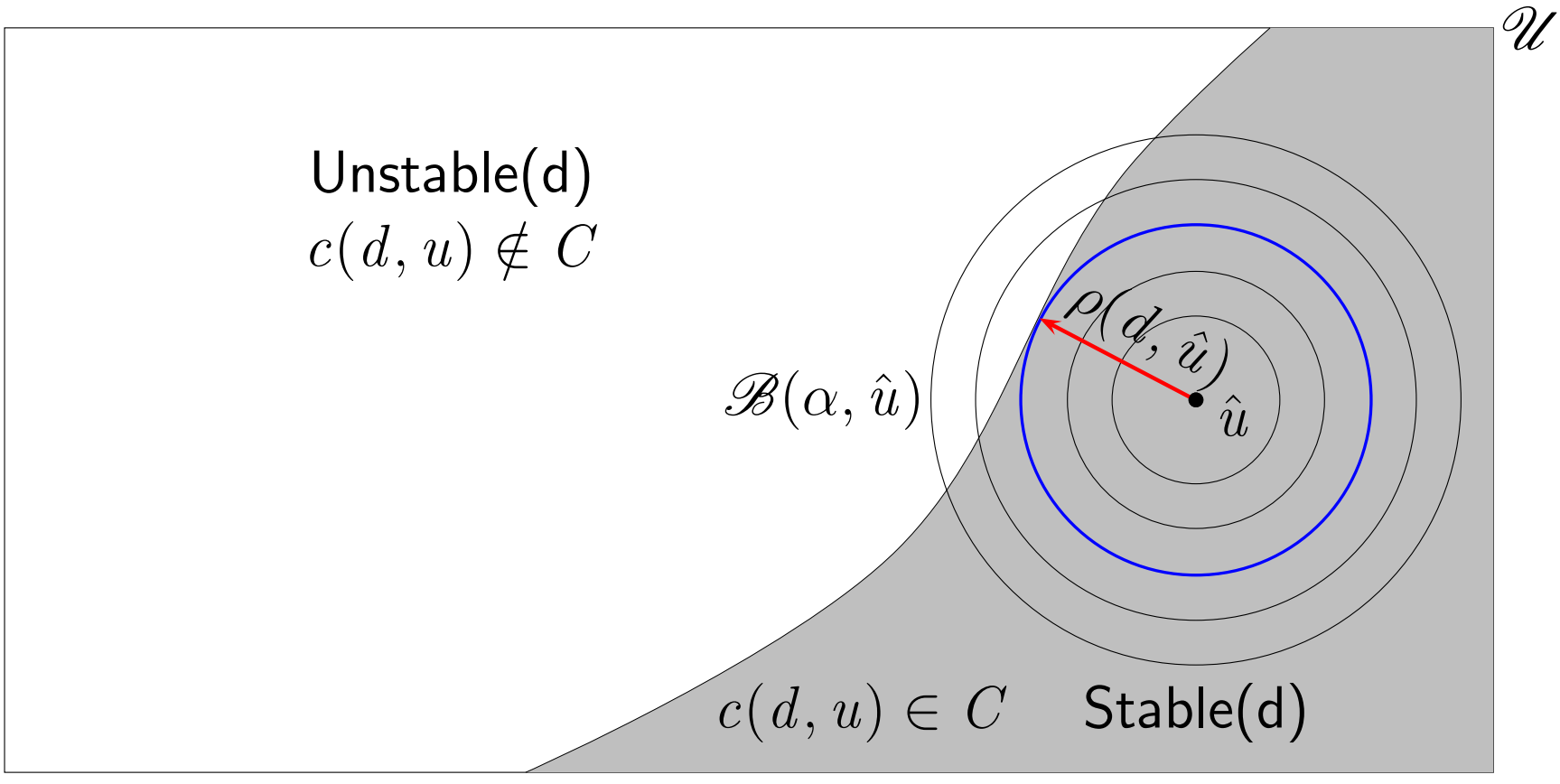
$\mathscr{B}(\alpha, \hat{u}) =$ ball of radius α centered at $\hat{u}$.

$c(d, u) \in C =$ stability requirement for decision $d \in D$.

Local Robustness

Radius of Stability (1960s)

$$\rho(d, \hat{u}) := \max\{\alpha \geq 0 : c(d, u) \in C, \forall u \in \mathscr{B}(\alpha, \hat{u})\}$$



Local Robustness

Radius of Stability (1960s)

$$\rho(d, \hat{u}) := \max\{\alpha \geq 0 : c(d, u) \in C, \forall u \in \mathscr{B}(\alpha, \hat{u})\}$$

Maximin Theorem (Sniedovich 2007, 2010)

The Radius of Stability model is a Maximin model:

$$\begin{aligned}\rho(d, \hat{u}) &:= \max\{\alpha \geq 0 : c(d, u) \in C, \forall u \in \mathscr{B}(\alpha, \hat{u})\} \\ &= \max_{\alpha \geq 0} \min_{u \in \mathscr{B}(\alpha, \hat{u})} \varphi(\alpha, u)\end{aligned}$$

$$\varphi(\alpha, u) := \begin{cases} \alpha & , \quad c(d, u) \in C \\ -\infty & , \quad c(d, u) \notin C \end{cases}$$

Local Robustness

Radius of Stability (1960s)

$$\rho(d, \hat{u}) := \max\{\alpha \geq 0 : c(d, u) \in C, \forall u \in \mathscr{B}(\alpha, \hat{u})\}$$

Invariance Theorem (Sniedovich 2007, 2010)

The radius of stability model is invariant with the uncertainty space $\mathscr{U}$ in that the results it generates are invariant with $\mathscr{U}$ as along as

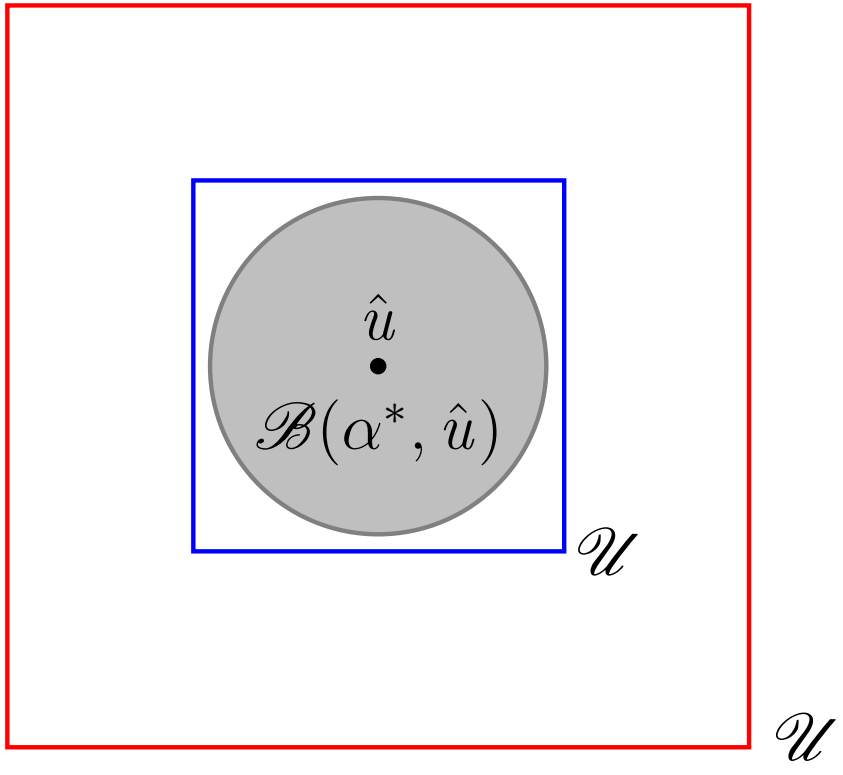
$$\mathscr{B}(\alpha^* + \varepsilon, \hat{u}) \subseteq \mathscr{U}$$

for some $\varepsilon > 0$, where $\alpha^* := \max_{d \in D} \rho(d, \hat{u})$.

Local Robustness

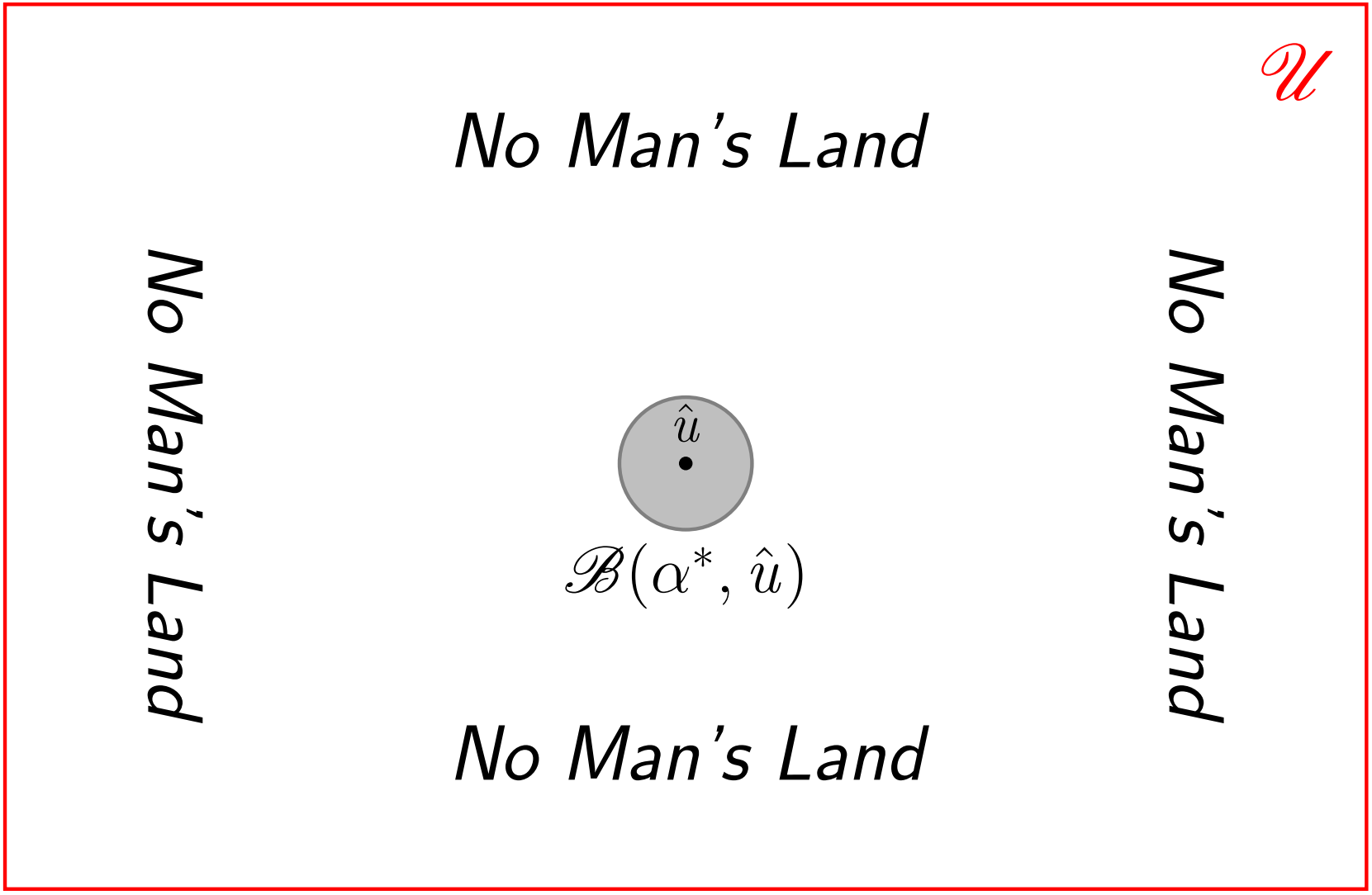
Invariance Theorem (Sniedovich 2007, 2010)

The radius of stability model is invariant with the uncertainty space $\mathscr{U}$ in that the results it generates are invariant with $\mathscr{U}$ as along as $\mathscr{B}(\alpha^* + \varepsilon, \hat{u}) \subseteq \mathscr{U}$



Local Robustness

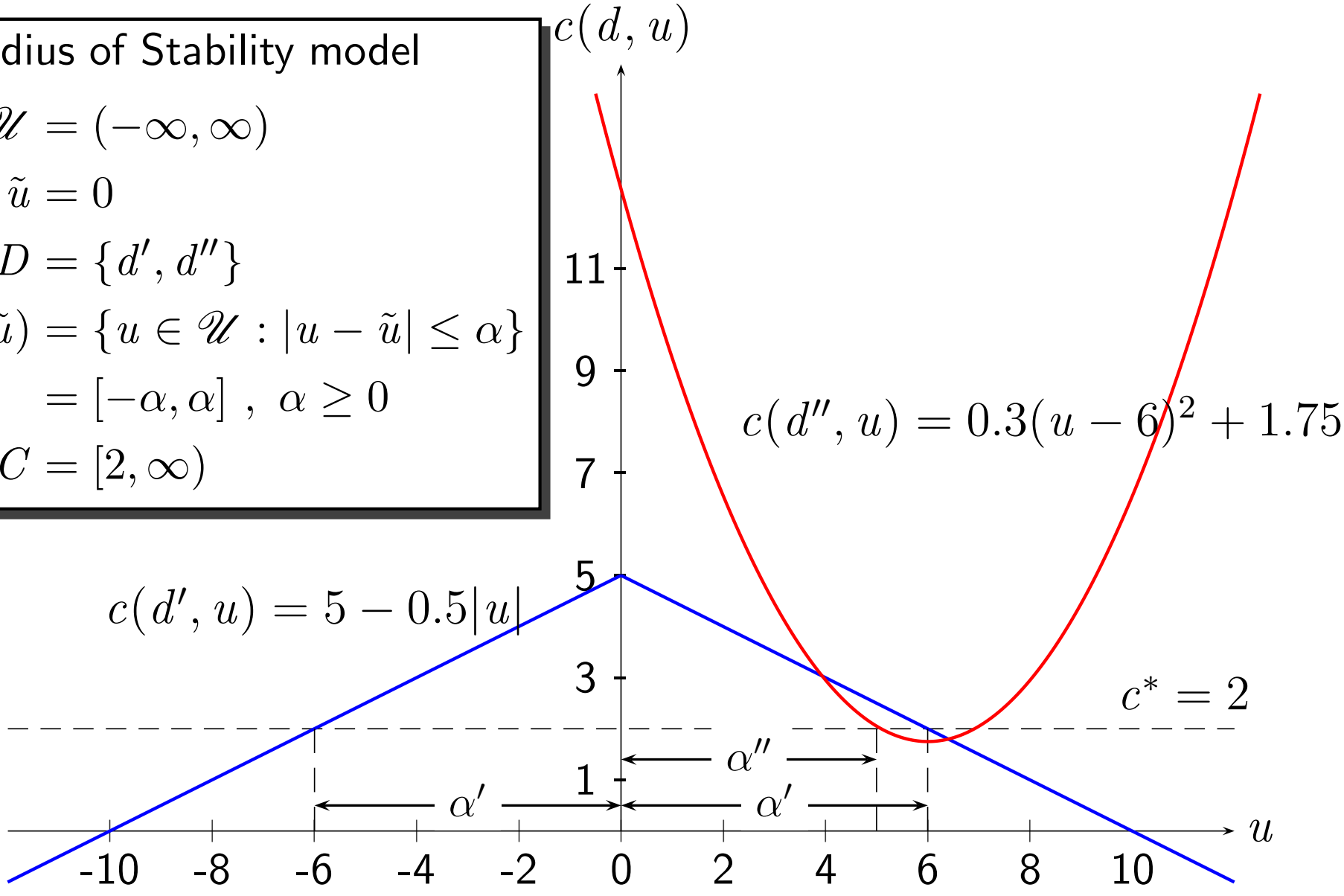
No Man's Land Syndrome



Example

Radius of Stability model

$$\begin{aligned}
\mathscr{U} &= (-\infty, \infty) \\
\tilde{u} &= 0 \\
D &= \{d', d''\} \\
\mathcal{B}(\alpha, \tilde{u}) &= \{u \in \mathscr{U} : |u - \tilde{u}| \leq \alpha\} \\
&= [-\alpha, \alpha] \ , \ \alpha \geq 0 \\
C &= [2, \infty)
\end{aligned}$$



The Campaign

- Launched at the end of 2006.
- Expected completion date: 11:59PM, January 1, 2012.
- Aim: to contain the spread of voodoo decision-making in Australia.
- Progress: Slow, but on course.
- Major issue: *You can't be a prophet in your own land!*
- More information:

info-gap.moshe-online.com

The Campaign: Example

You can't be a prophet in your own land!

2009 DEFRA (UK) Report, p. 75

More recently, Info-Gap approaches that purport to be non-probabilistic in nature developed by Ben-Haim (2006) have been applied to flood risk management by Hall and Harvey (2009). Sniedovich (2007) is critical of such approaches as they adopt a single description of the future and assume alternative futures become increasingly unlikely as they diverge from this initial description. The method therefore assumes that the most likely future system state is known a priori. Given that the system state is subject to severe uncertainty, an approach that relies on this assumption as its basis appears paradoxical, and this is strongly questioned by Sniedovich (2007).

The Campaign: Example

You can't be a prophet in your own land!

Rout et al. 2009, p.

Although info-gap theory is relevant for many management problems, two components must be carefully selected:the nominal estimate of the uncertain parameter, and the model of uncertainty in that parameter.If the nominal estimate is radically different from the unknown true parameter value, then the horizon of uncertainty around the nominal estimate may not encompass the true value, even at low performance requirements.Thus, the method challenges us to question our belief in the nominal estimate, so that we evaluate whether differences within the horizon of uncertainty are 'plausible'. Our uncertainty should not be so severe that a reasonable nominal estimate cannot be selected.

The Campaign: Example

Conceptual Hurdle

Role of an "estimate" under severe uncertainty

Severe Uncertainty

- The model is likelihood-free
- The uncertainty space is vast
- The point estimate can be substantially wrong

Point Estimate $\hat{u}$

- $\hat{u}$ is the most likely value of u
- Values of u become decreasingly unlikely as they deviate from $\hat{u}$
- The true value is most likely in the neighborhood of $\hat{u}$

Conclusions: OR perspective

- Decision-making under severe uncertainty is difficult.
- It is a thriving area of research/practice.
- The Robust Optimization literature is extremely relevant.
- The Decision Theory literature is extremely relevant.
- The Operations Research literature is very relevant.
- Severe uncertainty
- Black Swans
- New Nostradamuses
- Voodoo decision-making
- Challenges and Opportunities

Off the record

The Ten Natural Laws of Operations Analysis

Bob Bedow, *Interfaces* 7(3), p. 122, 1979

1. Ignore the problem and go immediately to the solution, that is where the profit lies.
2. There are no small problems only small budgets.
3. Names are control variables.
4. Clarity of presentation leads to aptness of critique.
5. Invention of the wheel is always on the direct path of a cost plus contract.
6. Undesirable results stem only from bad analysis.
7. It is better to extend an error than to admit to a mistake.
8. Progress is a function of the assumed reference system.
9. Rigorous solutions to assumed problems are easier to sell than assumed solutions to rigorous problems.
10. In desperation address the problem.

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