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# What exactly is wrong with Info-Gap? A Decision Theoretic Perspective

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# Abstract

- Info-Gap (Ben-Haim[2001, 2006]) is a young theory for decision-making under severe uncertainty.
- Info-Gap claims to be new and radically different from all current theories for decision making under uncertainty in that its uncertainty model is probability-free.
- We show that this is not so: Info-Gap's generic decision model is neither new nor radically different. It is a simple instance of the most famous model in classical decision making under severe uncertainty, namely Wald's [1945] Maximin model.
- We also show that Info-Gap's uncertainty model is fundamentally flawed: it does not tackle severe uncertainty – it simply and unceremoniously ignores it.
- FYI, Australia is an Info-Gap stronghold.

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# Maths Classification MA +18

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Principles

**5** Myths and Facts

#### 6 Conclusions

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An old legend has it that an ancient treasure is hidden in an Asian-Pacific island.



You are the head of the treasure hunt. How would you plan the operation?

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# Terminology



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# Certainty $\in$ Risk

#### Hence, the main issue is



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#### 1.2.3 Recipe

- Ignore the severe uncertainty.
- Focus on the very poor estimate you have.
- Conduct the analysis in the immediate neighborhood of the poor estimate.

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### Voodoo Decision-Making





- First encounter: An invitation to a seminar (3/8/03)
- Second encounter: Seminar (Ben-Haim, 2/9/03).
- Requests for comments on Info-Gap: 2/9/03 present.
- Informal critique: 3/9/03 present.
- Formal critique: 1/12/06 present.
- Campaign launch: 31/12/06.
- First feedback from Ben-Haim: (Friday! 13/4/07).
- Seminars: ASOR (1/12/06), ACERA (4/5/07), ORSUM (21/5/07), MS Colloquium (1/8/07)
- On the agenda:
  - Seminars
  - Honours thesis
  - Conference presentations
  - Articles
  - Book

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# What exactly is wrong with Info-Gap? A decision theoretic perspective

Key words:

- Exactly
- Decision Theoretic

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- A quick look at Info-Gap
- Classical Decision Theory for Decision Making Under Severe Uncertainty
- Info-Gap Revisited

# Self-Portrait

Info-gap decision theory is radically different from all current theories of decision under uncertainty. The difference originates in the modelling of uncertainty as an information gap rather than as a probability. The need for info-gap modeling and management of uncertainty arises in dealing with severe lack of information and highly unstructured uncertainty. Ben-Haim [2006, p. xii]

In this book we concentrate on the fairly new concept of information-gap uncertainty, whose differences from more classical approaches to uncertainty are real and deep. Ben-Haim [2006, p. 11]

#### **Obvious** Questions

Does Info-Gap substantiate these very strong claims?

Are these claims valid?

#### Not So Obvious Answers

- No, it does not.
- Ocertainly not.

Formal

Therefore, we shall do here and now – on the fly – what Info-Gap should have done seven years ago:

# Info-Gap

Analysis

Classical Decision Theory

Good news: should not take more than 5-10 minutes!

#### Summary of Results

There are serious gaps in Info-Gap. The following is a partial list:

- Info-Gap has grave misconceptions about the state of the art in decision-making under severe uncertainty.
- The generic Info-Gap decision model is a naive instance of the famous classical Maximin model (Wald, 1945).
- Info-Gap's uncertainty model is fundamentally flawed. It does not deal with severe uncertainty, it simply ignores it.
- Info-Gap is unsuitable for decision-making under severe uncertainty.
- There are other problematic issues with Info-Gap.

We shall examine only the main issues.

Generic Ir	nfo-Gap	Model			
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- Uncertainty region (set),  $\mathfrak{U}$ .
- A parameter u whose true value, u°, is unknown except that u° ∈ 𝔄.
- An estimate  $\tilde{u} \in \mathfrak{U}$  of  $u^{\circ}$ .
- A parametric family of nested regions of uncertainty, *U*(α, ũ) ⊆ 𝔅, α ≥ 0, of varying size (α), centered at ũ. That is, it is assumed that *U*(0, ũ) = {ũ} and that *U*(α, ũ) is non-decreasing with α, namely

$$\alpha'', \alpha' \in \mathbb{R}_+, \ \alpha'' > \alpha' \Longrightarrow \mathcal{U}(\alpha', \tilde{u}) \subseteq \mathcal{U}(\alpha'', \tilde{u})$$
 (1)

- Set of feasible decisions, Q.
- Reward function  $R: \mathbb{Q} \times \mathfrak{U} \to \mathbb{R}$ .
- Critical reward level,  $r_c \in \mathbb{R}$ .

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#### Robustness of a decision

$$\hat{\alpha}(q, r_c) := \max\left\{\alpha \ge 0 : r_c \le \min_{u \in \mathcal{U}(\alpha, \tilde{u})} R(q, u)\right\}$$
(2)

#### Optimal robustness

$$\hat{\alpha}(r_c) := \max_{q \in \mathbb{Q}} \hat{\alpha}(q, r_c)$$

$$= \max_{q \in \mathbb{Q}} \max \left\{ \alpha \ge 0 : r_c \le \min_{u \in \mathcal{U}(\alpha, \tilde{u})} R(q, u) \right\}$$
(4)

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#### Complete Model

$$\hat{\alpha}(r_c) := \max_{q \in \mathbb{Q}} \max\left\{ \alpha \ge 0 : r_c \le \min_{u \in \mathcal{U}(\alpha, \tilde{u})} R(q, u) \right\}$$
(5)

# Region of Severe Uncertainty, U



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#### Complete Generic Model

$$\hat{\alpha}(r_c) := \max_{q \in \mathbb{Q}} \max \left\{ \alpha \ge 0 : r_c \le \min_{u \in \mathcal{U}(\alpha, \tilde{u})} R(q, u) \right\}$$
(6)

#### Fundamental FAQs

1	Is this new?	Definitely not!
2	Is this radically different?	Definitely not!
3	Does it make <mark>sense</mark> ?	Definitely not!

So what is all this hype about Info-Gap ?!

Good question!

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Info-Gap	)				
		First	Impress	ion	
		Complet	e Generic	Model	
â	$(r_{a}) := m$	ax max $\int \alpha$	$> 0 : r_{c} <$	$\left\{\min_{R(a, y)}\right\}$	(7)

#### Observations

 $q \in \mathbb{Q}$ 

- This model does not deal with severe uncertainty, it simply and unceremoniously ignores it.
- The analysis is invariant with  $\mathfrak{U}$ : the same solution for all  $\mathfrak{U}$  such that  $\mathcal{U}(\hat{\alpha}(r_c), \tilde{u}) \subseteq \mathfrak{U}$ .

 $u \in \mathcal{U}(\alpha, \tilde{u})$ 

- This model is fundamentally flawed.
- This model advocates voodoo decision-making.

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First Impression							
Fool-Proof Re	cipe						

Step 1: Ignore the severe uncertainty.Step 2: Focus instead on the poor estimate and its immediate neighborhood.

Region of Severe Uncertainty



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Info-Gap					

#### First Impression

Region of Severe Uncertainty





Recall that this is voodoo decision making!

Info-Gap					
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Complete Generic Model

$$\alpha^* := \max_{q \in \mathbb{Q}} \max \left\{ \alpha \ge 0 : r_c \le \min_{u \in \mathcal{U}(\alpha, \tilde{u})} R(q, u) \right\}$$
(8)

## **Fundamental Flaw**



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# Classical Decision Theory



Eg.

620-262: Decision Making

A Simple	Problem	ı			
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Good morning Sir/Madam:

I left on your doorstep four envelopes. Each contains a sum of money. You are welcome to open any one of these envelopes and keep the money you find there.

Please note that as soon as you open an envelope, the other three will automatically self-destruct, so think carefully about which of these envelopes you should open.

To help you decide what you should do, I printed on each envelope the possible values of the amount of money (in Australian dollars) you may find in it. The amount that is actually there is equal to one of these figures.

Unfortunately the entire project is under severe uncertainty so I cannot tell you more than this.

Good luck!

Joe.

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So What	Do Yo	ou do?			
Fxam	hle				
EXam	510				
Env	elope	Possible	Amount (A	Australian dollars)	
1	E1		20, 10, 3	00,786	
1	$\Xi 2$	2,40000,1023	349,50000	00,99999999,5643543	32
1	$\Xi 3$		201,	202	
1	E4		20	0	

Vote!

Modeling	and Sol	ution			
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- What is a decision problem ?
- How do we model a decision problem?
- How do we solve a decision problem?

Decision	Tables	00000	
DEUSION	lables		

Think about your problem as a table, where

- rows represents decisions
- columns represent the relevant possible states of nature
- entries represent the associated payoffs/rewards/costs

Exa	Example						
_	Env		Possible Amount (\$AU)				
-	E1	20	10	300	786		
	E2	2	4000000	102349	500000000	56435432	
	E3	201	202				
	E4	200					

Classical decision theory distinguishes between three levels of uncertainty regarding the state of nature, namely

- Certainty
- Risk
- Strict Uncertainty

Terminology:

# Strict Uncertainty ≡ Severe Uncertainty ≡ Ignorance ≡ True Uncertainty ≡ Knightian Uncertainty



Classical decision theory offers two basic principles for dealing with severe uncertainty, namely

- Laplace's Principle (1825)
- Wald's Principle (1945)

Conceptually:



Bottom line: under severe uncertainty the estimate we have is a poor indicator of the true value it represents and is likely to be substantially wrong.

Assume that all the states are equally likely, thus use a uniform distribution function  $(\mu)$  on the state space and regard the problem as decision-making under risk.

#### Laplace's Decision Rule

$$\max_{d \in \mathbb{D}} \int_{s \in S_d} r(s, d) \mu(s) ds \qquad \text{Continuous case}$$
$$\max_{d \in \mathbb{D}} \frac{1}{|S_d|} \sum_{s \in S_d} r(s, d) \qquad \text{Discrete case}$$

Inspired by Von Neumann's [1928] Maximin model for 0-sum, 2-person games: Mother Nature is playing against you, hence apply the worst-case scenario. This transforms the problem into a decision-making under certainty.

Wald's Maximin Rule		
$\max_{\substack{d \in \mathbb{D}}}$	$\min_{s\in S_d}$	f(d,s)

Historical perspective: William Shakespeare (1564-1616)

The gods to-day stand friendly, that we may, Lovers of peace, lead on our days to age! But, since the affairs of men rests still incertain, Let's reason with the worst that may befall.

Julius Caesar, Act 5, Scene 1

Laplace v	vs Wald				
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Example							
	Env		Possi	ble Amou	nt (\$AU	)	
	E1	20	10	300	786		
	E2	2	4000	102349	50000	56435	
	E3	201	202				
	E4	200					

#### Example

	Env		Possik	ole Amoi	Laplace	Wald			
-	E1	20	10	300	786		279	10	
	E2	2	4000	10234	50000	56435	24134.2	2	
	E3	201	202				201.5	201	
	E4	200					200	200	

Laplace v	s Wald				
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ample									
Env		Possible Amount (\$AU) Laplace Wald							
E1	20	10	300	786		279	10		
E2	2	4000	10234	50000	56435	24134.2	2		
E3	201	202				201.5	201		
E4	200					200	200		
	$     Env \\     E1 \\     E2 \\     E3 \\     E4   $	Env         20           E1         20           E2         2           E3         201           E4         200	The second sec	ampleEnvPossible Amou $E1$ 2010 $E2$ 2400010234 $E3$ 201202 $E4$ 200 $$	Env       Possible Amount (\$AU         E1       20       10       300       786         E2       2       4000       10234       50000         E3       201       202       4000       10234       50000	ample         Env       Possible Amount (\$AU)         E1       20       10       300       786         E2       2       4000       10234       50000       56435         E3       201       202       4       4       4       4	ampleEnvPossible Amount (\$AU)Laplace $E1$ 2010300786279 $E2$ 2400010234500005643524134.2 $E3$ 2012021001000201.5 $E4$ 20010010001000200	ampleEnvPossible Amount (\$AU)LaplaceWald $E1$ 201030078627910 $E2$ 2400010234500005643524134.22 $E3$ 201202444200201 $E4$ 2004444200200	

Severe U	ncertaint	z <b>y</b>			
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#### Warning!

- For obvious reasons, methodologies for decision-making under severe uncertainty are austere.
- There are no miracles in this business.
- The essential difficulty is: how do you sample the uncertainty region?
- The best estimate we have is very poor and likely to be substantially wrong.
- If you are offered a methodology that is too good to be true,...it is too good to be true!

Myths an	d Facts				
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#### The following simple results will be very useful:

#### Theorem

$$\hat{\alpha}(q, r_c) := \max \left\{ \alpha \ge 0 : r_c \le \min_{u \in \mathcal{U}(\alpha, \tilde{u})} R(q, u) \right\}$$
(9)  
$$= \max_{\alpha \ge 0} \min_{u \in \mathcal{U}(\alpha, \tilde{u})} \alpha \cdot (r_c \preceq R(q, u))$$
(10)

#### where

$$a \leq b := \begin{cases} 1 & , a \leq b \\ 0 & , a > b \end{cases}$$
 (11)

A proof by inspection is immediate. Note the  $\min/\min$  issue.

Myths and	d Facts				
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#### Myth # 1

Classical decision theory does not offer probability-free approaches to decision-making under severe uncertainty.

#### Fact # 1

This is preposterous. Practically all introductory textbooks on decision theory discuss probability-free paradigms for decision-making under severe uncertainty. The most famous one is Wald's Maximin Model [1945].

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Example					

#### CHOICES An Introduction to Decision Theory Michael D. Resnik 1987

Chapter 1

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- 1-1 What is Decision Theory?
- 1-2 The Basic Framework
- 1-3 Certainty, Ignorance, and Risk
- 1-4 Decision Trees
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#### Chapter 2

Decisions Under Ignorance

- 2-1 Preference Ordering
- 2-2 The Maximin Rule
- 2-3 The Minimax Regret Rule
- 2-4 The Optimism-Pessimism Rule
- 2-5 The Principle of Insufficient Reason
- 2-6 Too many Rules?
- 2-7 An application in Social Philosophy

2-8 References

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#### Chapter 3

Decisions Under Risk: Probability

- 3-1 Maximizing Expected Values
- 3-2 Probability Theory
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#### Chapter 4

Decisions under Risk: Utility

4-1 Interval Utility Scales

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#### Myth # 2

Info-Gap's region of uncertainty is unbounded, therefore there is no worst case, and info-gap is not Maximin. (Ben-Haim [2005]).

#### Fact # 2

This is astounding!.

Comments:

- There can be a worst case even if the region of uncertainty is unbounded (eg sin(x), x ∈ ℝ).
- There is a worst case in all problems where Info-Gap yields a solution (Sniedovich [2006]).



#### 620-161: Introductory Mathematics

The most classical saddle point on  $\mathfrak{Planet}$  carth is associated with the unbounded region  $\mathbb{R}^2$  and the function

$$f(x,y) := x^2 - y^2$$

Its saddle point (x, y) = (0, 0) is the solution to the Maximin problem

$$z^* := \max_{y \in \mathbb{R}} \min_{x \in \mathbb{R}} \left\{ x^2 - y^2 \right\}$$

 $z^* := \max_{y \in \mathbb{R}} \min_{x \in \mathbb{R}} \left\{ x^2 - y^2 \right\}$ 



#### Myths and Facts

Introduction

Info-Gap

Ben-Haim [2001-2006] confuses various issues that are related to the structure of Info-Gap's uncertainty model:

Myths & Facts

Conclusions

$$\alpha(r_c) := \max_{q \in \mathbb{Q}} \max\left\{ \alpha \ge 0 : r_c \le \min_{u \in \mathcal{U}(\alpha, \tilde{u})} R(q, u) \right\}$$

#### • $\alpha$ is unbounded.

•  $\mathcal{U}(\alpha, \tilde{u})$  is (??????) unbounded.

Decision Theory

• R(q, u) is (?????) unbounded.

#### Example (Ben-Haim [2006])

• 
$$\mathcal{U}(\alpha, \tilde{u}) := \left\{ u \in [0, 1] : \left| \frac{u - \tilde{u}}{\tilde{u}} \right| \le \alpha \right\} , \ \alpha \ge 0$$

- $\alpha$  is unbounded.
- $\mathcal{U}(\alpha, \tilde{u}) \subseteq [0, 1]$  is bounded.
- There is definitely a worst case!

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#### Theorem

Info-Gap's uncertainty model attains a worst case.

#### Proof.

$$\hat{\alpha}(q, r_c) = \max_{\alpha \ge 0} G(\alpha) \cdot H(q, \alpha) , \ q \in \mathbb{Q},$$
(12)  
where  $G(\alpha) := \alpha , \ \alpha \ge 0$  (13)  
 $H(q, \alpha) := \min_{u \in \mathcal{U}(\alpha, \tilde{u})} (r_c \preceq R(q, u)) , \ \alpha \ge 0$  (14)  
 $a \preceq b := \begin{cases} 1 & , \ a \le b \\ 0 & , \ a > b \end{cases}$ (15)

Clearly,  $G(\alpha) \cdot H(q, \alpha) \in \{0, \alpha\}.$ 



$$\hat{\alpha}(q, r_c) := \max_{\alpha \ge 0} \min_{u \in \mathcal{U}(\alpha, \tilde{u})} \alpha \cdot (r_c \preceq R(q, u))$$



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$$\beta(q,\alpha) := G(\alpha) \cdot H(q,\alpha) = \alpha \cdot \min_{u \in \mathcal{U}(\alpha,\tilde{u})} (r_c \preceq R(q,u))$$
$$\in \{0,\alpha\}$$



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#### Myth # 3

Info-Gap is a new theory that is radically different from all current theories for decision-making under severe uncertainty (Ben-Haim [2001, 2006])

#### Fact # 3

Info-Gap's generic model is neither new nor radically different. It is a simple instance of Wald's Maximin model (Sniedovich [2006])



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#### Theorem (Sniedovich [2006])

Info-Gap's generic model is a simple Maximin Model.

#### Proof.

$$\alpha(r_c) := \max_{q \in \mathbb{Q}} \max\left\{ \alpha \ge 0 : r_c \le \min_{u \in \mathcal{U}(\alpha, \tilde{u})} R(q, u) \right\}$$
(16)  
$$= \max_{q \in \mathbb{Q}, \alpha \ge 0} \min_{u \in \mathcal{U}(\alpha, \tilde{u})} \alpha \cdot (r_c \le R(q, u))$$
(17)  
$$a \le b := \begin{cases} 1 & , a \le b \\ 0 & , a > b \end{cases}$$
(18)

#### Myth # 4

Info-Gap deals with severe uncertainty.

#### Fact # 4

Info-Gap does not deal with severe uncertainty. It ignores it. This involves:

- Replacing severe uncertainty by a very poor estimate of the parameter under consideration.
- Conducting standard maximin analysis in the neighborhood of this very poor estimate.



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Correctio	n:				

$$\alpha(r_c) := \max_{q \in \mathbb{Q}} \max\left\{ \alpha \ge 0 : r_c \le \min_{u \in \mathcal{U}(\alpha, \tilde{u})} R(q, u) \right\}$$

#### Info-Gap Interpretation

 $\hat{\alpha}(q,r_c):=$  robustness of decision q given the required reward  $r_c.$ 

#### Correct interpretation

 $\hat{\alpha}(q, r_c, \tilde{u}) :=$  robustness of decision q given the required reward  $r_c$ , in the neighborhood of the POOR estimate  $\tilde{u}$  that is likely to be SUBSTANTIALLY WRONG.

worst 
$$\tilde{u}$$
  
value  $\tilde{u}$   
of  $u$   
in  $\mathcal{U}(\alpha, \tilde{u})$ 

 $u^{\circ}$ rue value

Myth and	Facts				
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#### Observation

The Info-Gap analysis is invariant with the actual size of the total region of uncertainty,  $\mathfrak{U}$ . Surely, this makes no sense.

 $\widetilde{u}$   $\mathcal{U}(\alpha, \tilde{u})$  $\widetilde{u}$   $\mathcal{U}(\alpha, \tilde{u})$ worst worst value  $\alpha$ value  $\alpha$ of uof n in  $\mathcal{U}(\alpha, \tilde{u})$ in  $\mathcal{U}(\alpha, \tilde{u})$ 

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#### Complete Generic Model

$$\alpha^* := \max_{q \in \mathbb{Q}} \max \left\{ \alpha \ge 0 : r_c \le \min_{u \in \mathcal{U}(\alpha, \tilde{u})} R(q, u) \right\}$$
(19)

# Fundamental Flaw



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#### Myth # 5

Info-Gap generates robust solutions for decision-making problems under severe uncertainty.

#### Fact # 5

There is no reason to believe that under severe uncertainty the solutions generated by Info-Gap are robust (see explanation and counter examples in Sniedovich [2006]).

# Region of Severe Uncertainty, U





Note the analogy to the difference between local and global optimization.



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#### Small/Grand World Dilemma

Restricting attention to a small world was justified, Savage noted, if the resulting probabilities, utilities, and action recommendations agreed with the grand world ones. He analyzed conditions under which this was the case, but was uncomfortable that the conditions he derived were "incapable of verification without taking the grand world much too seriously."

Laskey and Lehner [1994, p. 1643]

As long as the small world model's predictions are reasonably accurate, the small world model will be a reasonable approximation to the large world.

Laskey and Lehner [1994, p. 1651]

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#### Comment

To obtain a robust solution under severe uncertainty you have to incorporate in the analysis a number of point estimates, making sure that they adequately represent the entire region of uncertainty,  $\mathfrak{U}$ .



See the Worst-Case Analysis and Robust Optimization literature for tips, guidelines and inspiration.

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Algorithms for Worst-case Design and Applications to Risk Management Rustem and Howe, [2002, p. xiii]

... If the forecaster tries to specify too many discrete forecasts, in an attempt to cover most possibilities, discrete minimax may yield too pessimistic strategies or even run into numerical, or computational, problems due to the resulting numerous scenarios. Similarly, as the upper and lower bounds on a range of forecasts get wider, to provide coverage to a wider set of possibilities, the minimax strategy may become pessimistic. Thus, scenarios have to be chosen with care, among genuinely likely values. The minimax strategy will then answer the legitimate question of what the best strategy should be, in view of the worst case ...

Myths an	d Facts				
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#### Myth # 6

Info-Gap is about the gap between what we know (poor estimate) and what we need to know (true value).

#### Fact # 6

Info-Gap is not about the gap between what we know (poor estimate) and what we need to know(true value). It is about ignoring the gap between what we know (poor estimate) and what we need to know (true value).

 $u^{\circ}$ 

true value

worst 
$$\tilde{u}$$
  
value  $\tilde{u}$   
of  $u$   
in  $\mathcal{U}(\alpha, \tilde{u})$ 

 $\tilde{u}$ 

Conclusio	ns				
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- Decision-making under severe uncertainty is difficult.
- It is a thriving area of research/practice.
- The Robust Optimization literature is extremely relevant.
- The Decision Theory literature is extremely relevant.
- The Operations Research literature is very relevant.
- Info-Gap's decision model is neither new nor radically different.
- Info-Gap's uncertainty model is fundamentally flawed and unsuitable for decision-making under severe uncertainty.
- Info-Gap exhibits a severe information-gap about the state of the art in decision-making under severe uncertainty.
- It is time to reassess the use of Info-Gap in Australia.



Join the Campaign!

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#### Myth # 7

Satisficing is better than optimizing.

#### Fact # 7

Any satisficing problem can be formulated as an (equivalent ) optimization problem (Sniedovich [2006]).

#### Comments:

- Strictly and bluntly speaking, the assertion that satisficing is superior to optimizing is nonsensical.
- What is important is what you optimize and what you satisfice.
- Indeed, Info-Gap optimizes robustness!!!

#### Theorem (Sniedovich [2006])

Any satisficing problem can be expressed as an equivalent optimization problem.

#### Proof.

Let *I* denote the universal indicator function:

$$I_X(x) := \begin{cases} 1 & , \ x \in X \\ 0 & , \ x \notin X \end{cases}$$
(20)

Then clearly,

$$x \in X \subseteq X' \iff x = \arg \max_{x \in X'} I_X(x)$$
 (21)

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#### Example

You win a game (AU\$5,000,000) if you select an action  $q \in \mathbb{Q}$  such that  $17 \leq \sigma(q) \leq 21$ , where  $\sigma$  is a given real-valued function on  $\mathbb{Q}$ .

#### Problem: Find a $q \in \mathbb{Q}$ such that $17 \leq \sigma(q) \leq 21$

This is typical satisficing model. Note that, in general, to win the game you do not necessarily optimize the score  $\sigma(q)$  over  $q \in \mathbb{Q}$ . The following is an equivalent optimization model:

$$\max_{q \in \mathbb{Q}} 5w(q)$$
(22)  
$$w(q) := \begin{cases} 1 & , \ 17 \le \sigma(q) \le 21 \\ 0 & , \ otherwise \end{cases}$$
(23)

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Myth # 8

Robustness is more important than performance.

#### Fact # 8

This is a counter-productive and obviously invalid generalization. This assertion is particularly nonsensical in the framework of a methodology calling for a Pareto trade-off between robustness and performance.

Info-Gap Generic Model

$$\hat{\alpha}(r_c) := \max_{q \in \mathbb{Q}} \max \left\{ \alpha : r_c \le \min_{u \in \mathcal{U}(\alpha, \tilde{u})} R(q, u) \right\}$$
(24)